TIME SERIES ANALYSIS & FORECASTING

BASIC CONCEPTS AND FORMULA

Basic Concepts

1. Time Series Analysis

The term 'Time Series' means a set of observations concurring any activity against different periods of time. In order to describe this flow of economic activity, the statistician uses a time series.

2. Examples of Time Series Data

Following are few examples of time series data:

a) Profits earned by a company for each of the past five years.

b) Workers employed by a company for each of the past 15 years.

c) Number of students registered for CA examination in the institute for the past five years.

d) The weekly wholesale price index for each of the past 30 week.

e) Number of fatal road accidents in Delhi for each day for the past two months.

3. Components of a Time Series:

A time series may contain one or more of the following four components:

1. Secular trend (T): (Long term trend) It is relatively consistent movement of a variable over a long period.

2. Seasonal variation (S): Variability of data due to seasonal influence.

3. Cyclical variation (C): Recurring sequence of points above and below the trend line lasting over more than one year.

4. Irregular variation (I): (random movements) Variations due residual factors that accounts for deviations of the actual time series values from those expected, given the effect of trend, seasonal and seasonal components. Example, erratic movements that do not have any pattern and are usually caused by unpredictable reason like earthquake, fire etc.

4. Approaches for the Relationship amongst Components of a Time Series

There are two approaches for the relationship amongst these components.

(a) Y = T×S×C×I (multiplicative model)

(b) Y = T+S+C+I (additive model)

Note: In multiplicative models S,C and I indexes are expressed as decimal percents

Where Y is the result of the four components.

5. Trend

The trend is the long-term movement of a time series. Any increase or decrease in the values of a variable occurring over a period of several years gives a trend. If the values of a variables remain statutory over several years, then no trend can be observed in the time series.

6. Methods of Fitting a Straight Line to a Time Series

i. Free hand method,

ii. The method of semi-averages,

iii. The method of moving averages

iv. The method of least squares.

7. Methods of Finding Short Period Variations

Other Methods of finding short period variations

7.1 Simple Average:

Simple Average: The method is very simple: average the data by months or quarters or years and them calculate the average for the period. Then find out, what percentage it is to the grand average.

$$\text{Seasonal Index} = \frac{\text{MonthlyorQuaterlyAverage}}{\text{GrandAverageofthemonthsorthequaters}} \text{x100}$$

Same results are obtained if the totals of each month or each quarter are obtained instead of the average of each month or each quarter.

7.2 Ratio-to-Trend Method

This method is an improvement over the previous method because this assumes that seasonal variation for a given month is a constant fraction of trend. This method presumably isolates the seasonal factor in the following manner:

$$S \times C \times I = \frac{T \times S \times C \times I}{T}$$

Random elements (I) are supposed to disappear when the ratios are averaged. Further, a carefully selected period of years used in computation is expected to eliminate the influence of cyclical fluctuations (C).

8. Deseasonalization

The process of eliminating seasonal fluctuations or deseasonalization of data consists of dividing each value in the original series by the corresponding value of the seasonal index.

9. Forecasting

Time series forecasting methods involve the projection of future values of a variable based entirely on the past and present observation of that variable.

10. Various Forecasting Methods Using Time Series.

10.1 Mean Forecast

The simplest forecasting method in which for the time period t. we forecast the value of the series to be equal to the mean of the series. This method is not adequate as trend effects and the cyclical effects are not taken into account in this.

10.2 Naïve forecast

In this method, by taking advantage of the fact that there may be high correlation between successive pairs of values in a time series, we forecast the value, for the time period t, to-be equal to the actual value observed in the previous period t that is, time period (t – 1):

$$y_t = y_{t-1}$$

10.3 Linear Trend Forecast

In this method, a linear relationship between the time and the response value has been found from the linear relationship.

$$y_t = a + bX$$

where X will be found from the value of t and a and b are constants.

10.4 Non-linear Trend Forecast

In this method, a non-linear relationship between the time and the response value has been found again by least-squares method. Then the value, for the time period t , will be calculated from the non-linear equation . i.e.,

$$y_t = a + bX + cX^2$$

where X-value will be calculated from the value of t.

10.5 Forecasting will Exponential Smoothing

In this method, the forecast value for the time period t is found using exponential smoothing of time series. Specifically, at the time period t.

$$y_t = y_{t-1} + \alpha(y_t - y_{t-1})$$

where the forecasted value for time period t + 1 ;

y_{t-1}= the forecasted value for time period t.:

y_t=the observed value for time period t.

Question 1

What is trend? What are the various methods of fitting a straight line to a time series?

Answer

Trend is the long term movement of a time series. Any increase or decrease in the values of a variable occurring over a period of several years gives a trend.

The various methods of fitting a straight line to a time series are:

(i) Free hand method.

(ii) The method of semi-averages.

(iii) The method of moving averages.

(iv) The method of least squares.

Question 2

Name the various methods of fitting a straight line to a time series and briefly explain any two of them.

Answer

The various methods of fitting a straight line are:

(i) Free hand method

(ii) Semi-average

(iii) Moving average

(iv) Least square

Freehand method:

First the time series figures are plotted on a graph. The points are joined by straight lines. We get fluctuating straight lines, through which an average straight line is drawn. This method is however, inaccurate, since different persons may fit different trend lines for the same set of data.

Method of Semi Averages:

The given time series is divided into two parts, preferably with the same number of years. The average of each part is calculated and then a trend line through these averages is filled.

Moving Average Method:

A regular periodic cycle is identified in the time series. The moving average of n years is got by dividing the moving total by n. The method is also used for seasonal and cyclical variation.

Method of Least Squares:

The equation of a straight line is Y = A + b X, where X is the time period, say year and Y is the value of the item measured against time, a is the Y intercept and b, the co-efficient of X, indicating the slope of the line. To find a and b, the following 'normal' equations are solved.

$$\sum Y = an + b \sum X$$

$$\sum XY = a \sum X + b \sum X^2$$

Where n is the no. of observation in the series or n = no. of data items.

Question 3

Apply the method of link relatives to the following data and calculate seasonal indices.

			Quarterly Figures		
Quarter	1995	1996	1997	1998	1999
I	6.0	5.4	6.8	7.2	6.6
II	6.5	7.9	6.5	5.8	7.3
III	7.8	8.4	9.3	7.5	8.0
IV	8.7	7.3	6.4	8.5	7.1

Answer

Calculation of seasonal indices by the method of link relatives.

Year	Quarter			
	I	II	III	IV
1995	–	108.3	120.0	111.5
1996	62.1	146.3	106.3	86.9
1997	93.2	95.6	143.1	68.8
1998	112.5	80.6	129.3	113.3
1999	77.5	110.6	109.6	88.8

Arithmetic average = $\frac{345.4}{4} = 86.35$ $\frac{541}{5} = 108.28$

$\frac{608.3}{5} = 121.66$ $\frac{469.3}{5} = 93.86$

Chain relatives 100 $\frac{100 \times 108.28}{100} = 108.28$

$\frac{121.66 \times 108.28}{100} = 131.73$

$\frac{93.86 \times 131.73}{100} = 123.65$

Corrected chain relatives 100 108 – 1.675 = 106.605

131.73 – 3.35 = 128.38

123.64 – 5.025 = 118.615

Seasonal indices $\frac{100 \times 100}{113.4}$ $\frac{106.605}{113.4} \times 100$ $\frac{128.38}{113.4} \times 100$

$\frac{118.615}{113.4} \times 100$

= 88.18 = 94.01 = 113.21 = 104.60

The calculation in the above table are explained below:

Chain relative of the first quarter (on the basis of first quarter = 100)

Chain relative of the first quarter (on the basis of the last quarter)

$$= \frac{86.35 \times 123.64}{100} = 106.7$$

The difference between these chain relatives = 106.7 – 100 = 6.7

Difference per quarter $= \frac{6.7}{4} = 1.675$

Adjusted chain relatives are obtained by subtracting 1 × 1.675, 2 × 1.675, 3 × 1.675 from the chain relatives of the 2nd, 3rd and 4th quarters respectively.

Average of corrected chain relatives

$$= \frac{100 + 106.605 + 128.38 + 118.615}{4} = \frac{453.6}{4} = 113.4$$

Seasonal variation index $= \frac{\text{Correct chain relatives}}{113.4} \times 100$

Question 4

The following table relates to the tourist arrivals during 1990 to 1996 in India:

Years :	1990	1991	1992	1993	1994	1995	1996
Tourists arrivals: (in millions)	18	20	23	25	24	28	30

Fit a straight line trend by the method of least squares and estimates the number of tourists that would arrives in the year 2000.

Answer

Fitting straight line Trend by the Method of Least squire

Year	Tourist Arrivals (in millions) Y	X	XY	X^2
1990	18	-3	-54	9
1991	20	-2	-40	4
1992	23	-1	-23	1
1993	25	0	0	0
1994	24	1	24	1
1995	28	2	56	4
1996	30	3	90	9

N = 7 $\Sigma y = 168$ $\Sigma x = 0$ $\Sigma xy = 53$ $\Sigma x^2 = 28$

The equation of the straight line trend is:

Y = a + b*x*

Since $\sum x = 0, a = \frac{\sum y}{N} = \frac{168}{7} = 24$

And $b = \frac{\sum xy}{\sum x^2} = \frac{53}{28} = 1.893$

Hence Y = 24 + 1.893*x*

Estimated Number of tourists that would arrive in 2000

Y = 24 + 1.893 (7) = 24 + 13.251 = 37.251 million.

EXERCISE

Question 1

Below are given the figures of production (in thousand quintals) of a sugar factory.

Year	Production (thousand quintals)
1993	77
1995	88
1996	94
1997	85
1998	91
1999	98
2002	90

(i) Fit a straight line by the 'least squares' method and tabulate the trend values.

(ii) Eliminate the trend. What components of the series are thus left over?

(iii) What is monthly increase in the production of sugar?

Answer

(i) equation of straight line trend is Y = 88.803 + 1.38 X

(ii) After eliminating the trend we are left with cyclical and irregular variations.

(iii) The monthly increase in the production of sugar is

b/12, i.e. 1.38 / 12 = 0.115 thousand quintal.

Question 2

Calculate 5 yearly and 7 yearly moving averages for the following data of the numbers of commercial and industrial failure in a country during 1987 to 2002.

Year	No. of failures
1987	23
1988	26
1989	28
1990	32
1991	20
1992	12

1993	12
1994	10
1995	9
1996	13
1997	11
1998	14
1999	12
2000	9
2001	3
2002	1

Also plot the actual and trend values on a graph.

Answer

Calculation of 5 – yearly and 7 – yearly moving Averages

Year	5 – yearly moving average	7 – yearly moving average
1987	–	–
1988	–	–
1989	25.8	–
1990	23.6	21.9
1991	20.8	20.0
1992	17.2	17.6
1993	12.6	15.4
1994	11.2	12.4
1995	11.0	11.6
1996	11.4	11.6
1997	11.8	11.1
1998	11.8	10.1
1999	13.8	9.0
2000	7.8	–
2001	–	–
2002	–	–