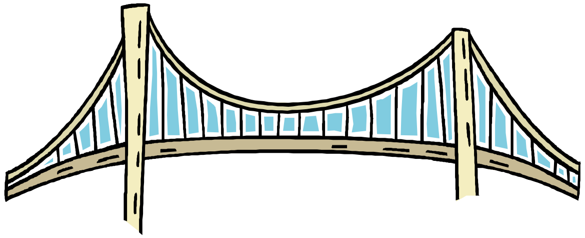
Continuous Improvement Toolkit

Probability Distributions



Managing Risk
PDPC
FMEA
RAID Logs
Fault Tree Analysis
Risk Assessment*
Traffic Light Assessment
Deciding & Selecting
Pros and Cons
Importance-Urgency Mapping
Break-even Analysis
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Force Field Analysis
Voting
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Decision Tree
TPN Analysis
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Prioritization Matrix
Kano Analysis
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RACI Matrix
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Gap Analysis
Hoshin Kanri
Kaizen
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Tree Diagram**
Standard work
Identifying & Implementing Solutions***
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Time Value Map
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Photography
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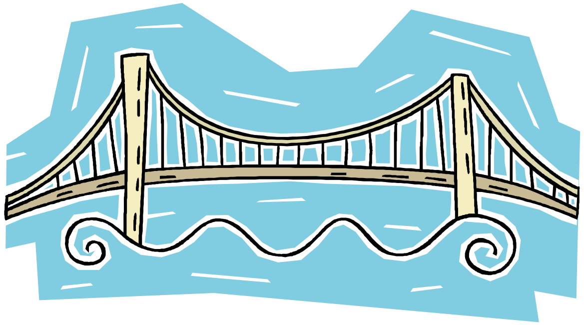
Nominal Group Technique
Mind Mapping*
Value Stream Mapping
SIPOC
Measles Charts
Surveys
Affinity Diagram
Attribute Analysis
Flow Process Chart
Process Mapping
Data Collection
Critical Incident Technique
Lateral Thinking
Visioning
Flowcharting
Service Blueprints
Observations
Creating Ideas**
Designing & Analyzing Processes

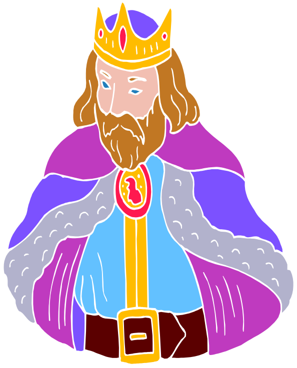
- Probability Distributions

- Probability distributions provide the bridge between descriptive and inferential statistics.
- They are mathematical models that we use to model real-life data distribution.
- Once we have found the appropriate model, we can use it for **prediction purposes**.

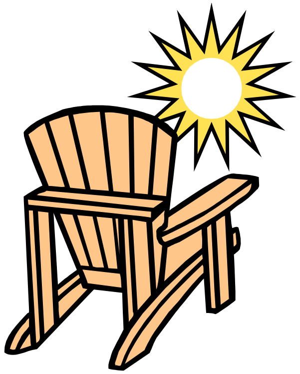


- Probability Distributions

- Probability runs on a scale of 0 to 1.
- A probability of 0 means ‘it will definitely not happen’.
  - **Example:** what is the probability that you will be the king of England?

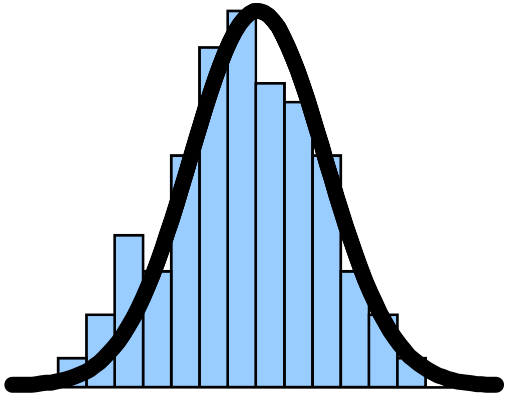


- A probability of 1 means ‘it is certain to happen’.
  - **Example:** what is the probability that the sun will rise tomorrow?



- Probability Distributions

- When dealing with process improvement, we generally have only **samples** to work with.
- A sampling distribution is the probability distribution of a given statistic based on a random sample.
- To draw conclusions from sample data, we compare values obtained from our sample with the theoretical values obtained from the probability distribution.
- We need to be sufficiently confident before we take any decision.
- This “Confidence level” is often set at 95% or 99%.



- Probability Distributions

Probability Distributions Types - Discrete Distributions:

- Deals with data that can take specific values.
- Count and Attribute data worlds.
- Two common discrete probability distributions: Binomial & Poisson.
- **Examples:**
  - The number of defective items in a sample.
  - The number of defects found in a single product.

Binomial

Poisson

Probability Distributions Types - Continuous Distributions:

- Relate to data which can take any value.
- There are a number of different distribution models and shapes.
- The commonest is the Normal distribution.
- The Exponential and Weibull distributions are widely used in the field of reliability engineering.

F

Uniform

Weibull

Student's T

Normal

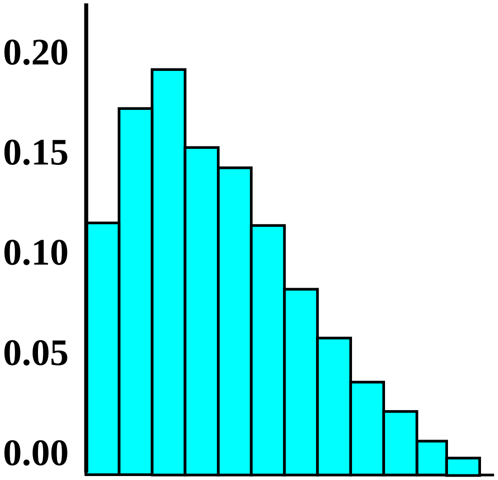
Exponential

- Probability Distributions

The Binomial Distribution:

- Used for data which can only take on of two values: pass/fail, yes/no, etc.
- **Example:**
  - Taking 15 samples from a large batch which is say 4% defective.

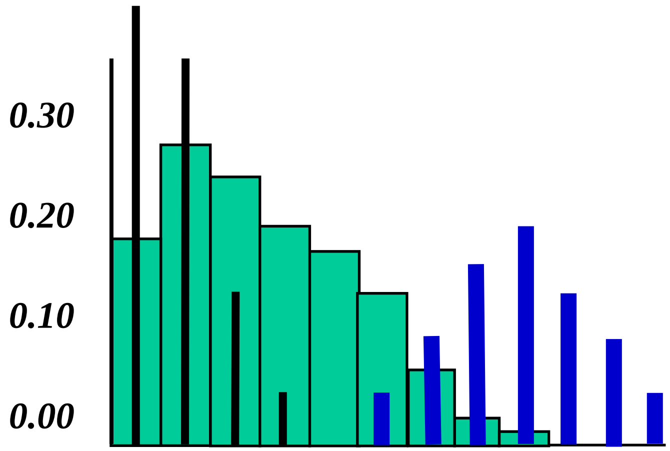
$$P(r) = \binom{n}{r} p^{r}(1-p)^{n-r}$$



- Probability Distributions

The Poisson Distribution:

- Used for data where there may be any number of defects in a given item.
- There must be a defined interval or area of opportunity (e.g. per sample).
- **Examples:**
  - The number of incoming calls per hour to a call center.
  - The number of failures per month for a specific equipment.
  - The number of accident per year in a factory.
- The exact shape depends solely on the value of λ.

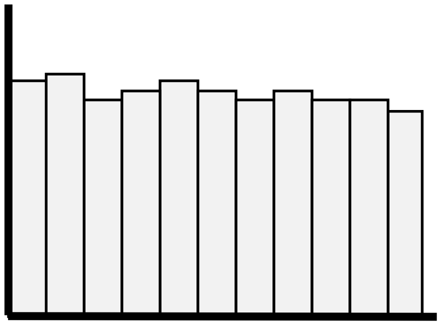


$$P(r) = \frac{\lambda^{r} e^{-x}}{r!}$$

- Probability Distributions

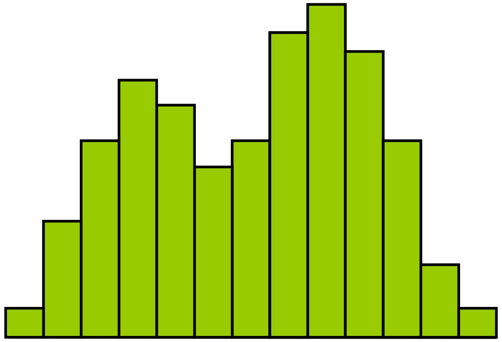
The Uniform Distribution:

- Describes variables that have a constant probability.
- Does not occur often in nature, but it is important as a reference distribution.
- An example of the discrete uniform distribution is throwing a dice.
- The possible values are 1, 2, 3, 4, 5, 6, and each time the dice is thrown the probability of a given score is 1/6.



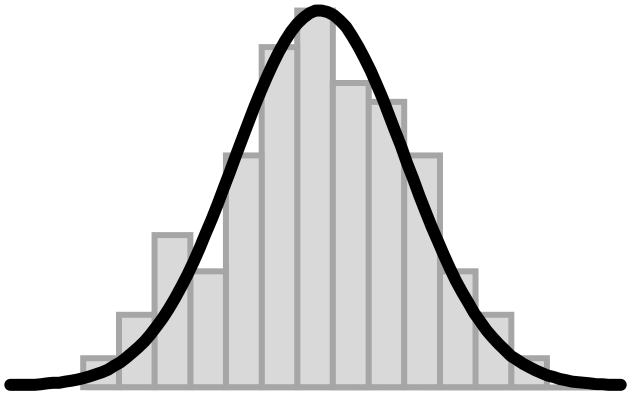
The Bimodal Distribution:

- Continuous probability distribution with two different modes.
- If observations are taken from different populations, multi-modality may result.
- **Example:**
  - Taking samples from two shifts.
  - The operators may setup the machine up differently.
- A **multimodal distribution** is a continuous probability distribution with two or more modes.



The Normal Distribution:

- A Normal distribution is a commonly occurring distribution.
- Natural phenomena, physical quantities and industrial processes follow approximately the Normal distribution.
- Often used in the natural and social sciences.
- If the normal distribution is applicable, it can be used to estimate future process performance.
- **Examples:**
  - Product thickness and weight.
  - Water purity and temperature.
  - Dimensions and test results.



- Probability Distributions

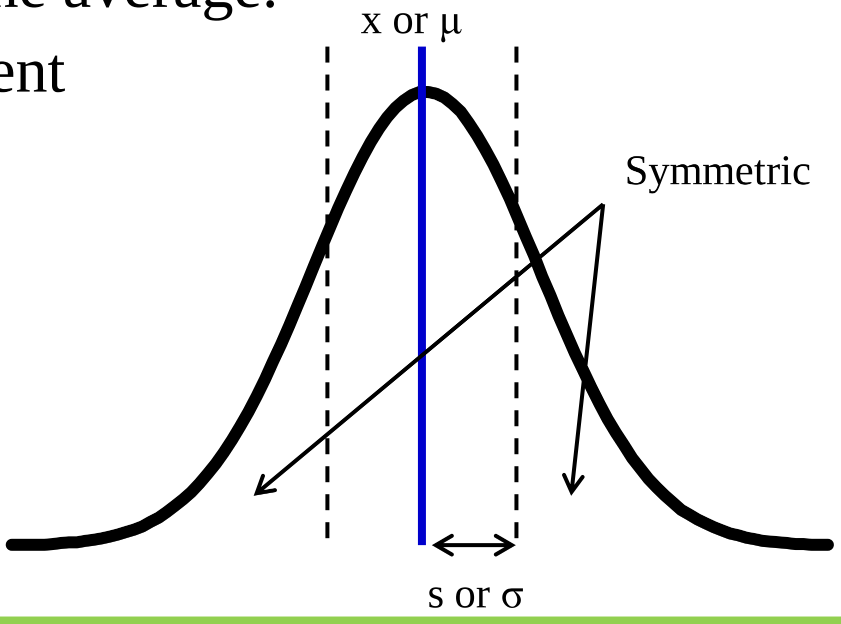
The Normal Distribution:

- It is symmetrical with most of the results in the middle and fewer toward the extremes.
- Fully defined by its mean and standard deviation.
- The peak of the curve represents the average.
- The spread of the curve is equivalent to six times the standard deviation of the process.

Symmetrical

Bell shaped

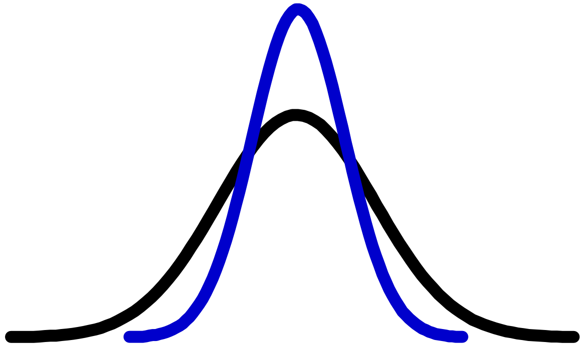
Centred on the mean



- Probability Distributions

The Normal Distribution:

- Uses a continuous curve to describe probabilities.
- The normal curve is based on the average and standard deviation of the histogram.
- A wider-flatter curve demonstrates more variation in the process.
- In theory, the normal distribution never ends but in practice, most results fall within +/– 3 Sigma.

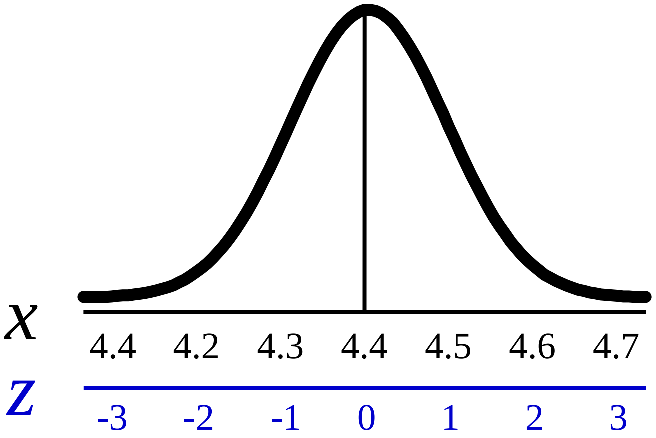


- Probability Distributions

The Standard Normal Distribution:

- Has a mean = zero, and a standard deviation = 1.
- Any normally distributed data can be converted to standard normal distribution.
- For any point 'x', the 'Z-value' is the number of standard deviations of that point from the mean.
- The area between two values gives the proportion of results that will occur between the two points.
- This calculation can give us a sense of the probability to make inferences about a population.

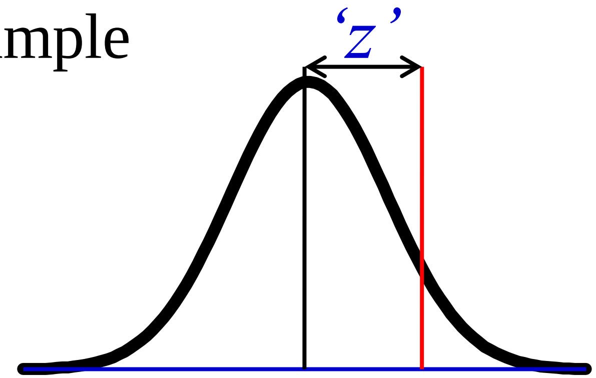
$$Z = \frac{x - \text{Average}}{\text{STDev}}$$



- Probability Distributions

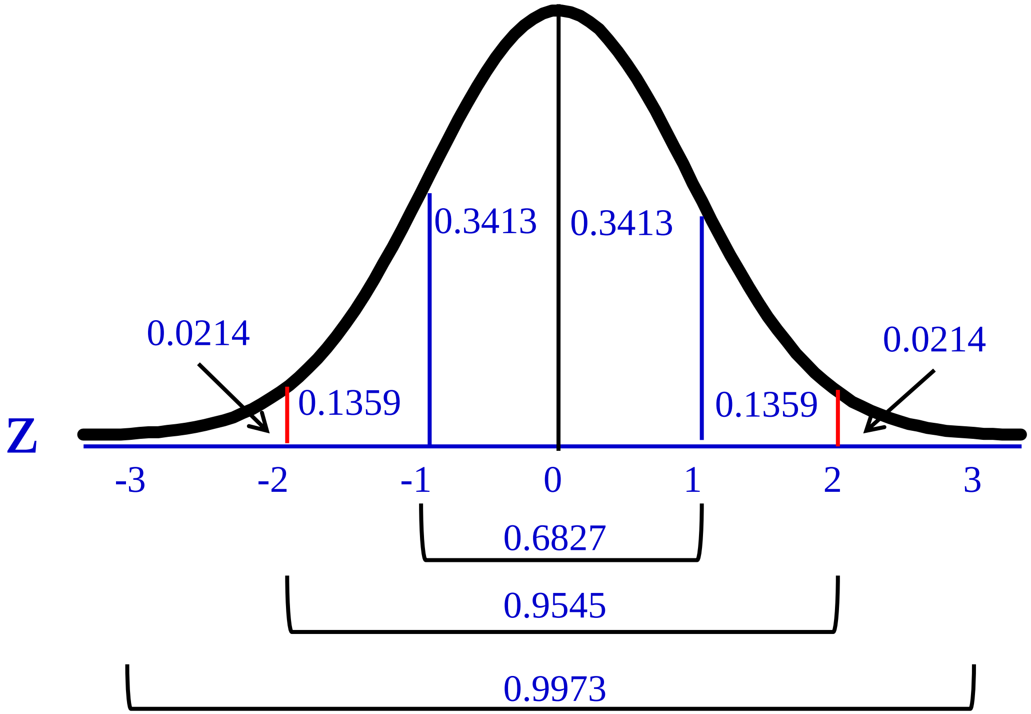
The Z-table:

- The complex shape of the normal curve has been converted into a mathematical table called the Z-Table.
- Used to find the probability that a statistic is observed below, above, or between values on the standard normal distribution.
- It is a common practice to convert a normal to a standard normal and then use the standard normal table to find probabilities.
- Using the Z-table, we can determine for example the proportion of outputs greater that the process average.



- Probability Distributions

- The Z-value measures the sample distance from the population mean, calculated in standard deviations.

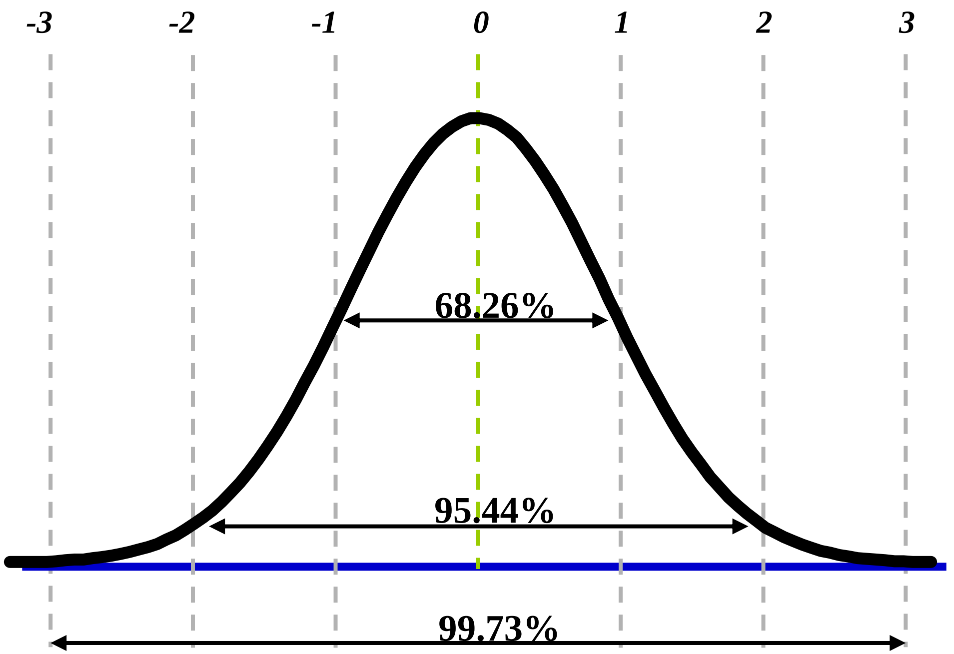


The total area under the normal curve equals to 1

- Probability Distributions

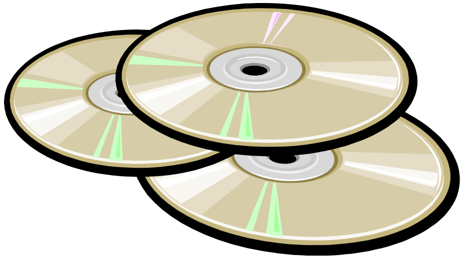
- If the normal distribution is applicable, it can be used to estimate future process performance.

For a Normally distributed output, 68% of the results will be within +/– Sigma of the average



- Probability Distributions

Example:

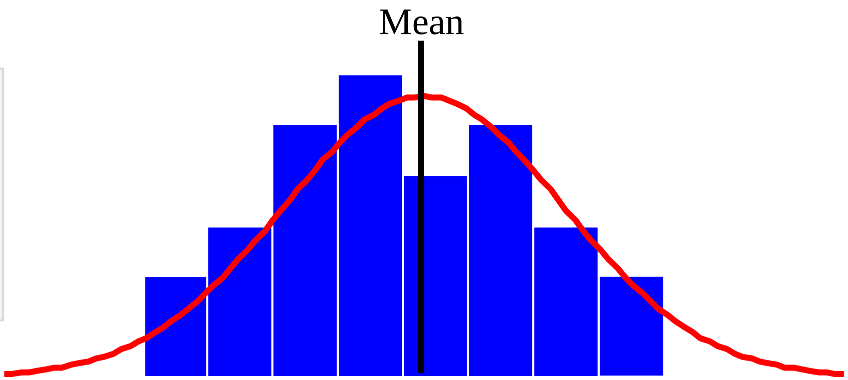


- A test run of **100** CDs has been made and the thickness results has the average of **1.35mm** and a standard deviation of **0.06**.
- None of the tested CDs were above the upper specification limit of **1.5mm**. However the Normal curve on the histogram still extends beyond.
- This suggests that over the long-term some CDs will be oversize.
- Given that the Normal distribution can be used, what is the probability that we will have defected CDs in the long term?
- The Z-table predicts the area under the curve to be 0.6%.
- This is different and a better prediction than the 0% predicted earlier.

- Probability Distributions

- **Several tools are available to help test data for normality:**
  - Histograms.
  - Normal probability plots.
  - Anderson-Darling normality test.

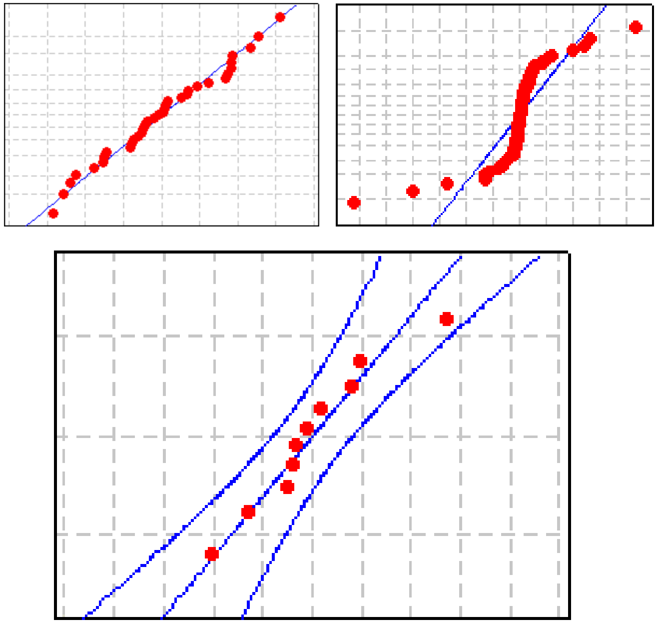
Histograms are efficient graphical methods for describing the distribution of data.



- Probability Distributions

Probability Plots:

- Provide a more decisive approach for deciding if a data set fits the normal distribution.
- Constructed in a way that the points will fall in a straight line if they fit the distribution question (e.g. Normal).
- A Normal distribution will form a straight line that falls between the 95% the CI limits.

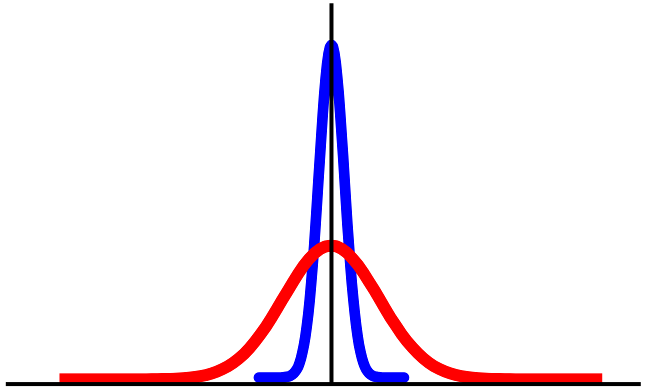


Normal Probability Plots

- Probability Distributions

Normal Distribution Other Characteristics:

- **Skewness:**
  - It is a measure of departure from symmetry.
  - Distributions may be skewed to the right (positive skew) or to the left (negative skew).
- **Kurtosis:**
  - It is a measure of the extent to which a distribution is flat-topped or peaked.
  - A positive value indicates a more peaked distribution than Normal.



- Probability Distributions

Further Considerations:

- **Left skewed distributions** may occur where the process has a physical upper limit or only has an upper specification limit.
- **Example:** the arrival time of job application since most job vacancies have a deadline and most applications arrive just before the deadline.
- **Right skewed distributions** are typically found when measuring time.
- This is because there is usually natural lower limit to how fact a process can be completed.
- **Examples:** The time for processing a job application.

- Probability Distributions

Further Considerations:

- It is important to understand the Normal distribution so that tools and techniques can be applied in a statistical valid manner.
- **Where the Normal distribution is important:**
  - **In Capability Analysis:** to make prediction about the process capability over the long term.
  - **In SPC:** certain charts require the data to be Normally distributed.
  - **In Hypothesis Testing:** many tests are based on the assumption that the data is Normally distributed.
  - **In Regression:** if the residual errors are Normally distributed, this indicates the regression model fits the data well.
  - **In Confidence Intervals and Design of Experiments.**