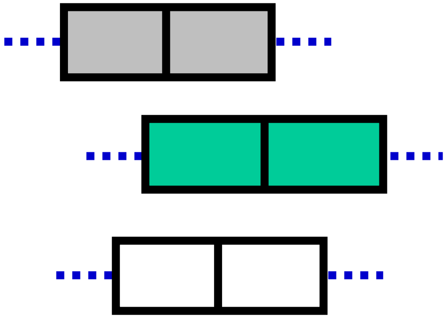
Continuous Improvement Toolkit

ANOVA




Managing Risk
PDPC
FMEA
RAID Logs
Fault Tree Analysis
Risk Assessment*
Traffic Light Assessment
Deciding & Selecting
Pros and Cons
Importance-Urgency Mapping
Break-even Analysis
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RACI Matrix
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Identifying & Implementing Solutions***
Simulation
TPM
Mistake Proofing
Pull Systems
JIT
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Work Balancing
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Bottleneck Analysis
Visual Management
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Observations
TRIZ***
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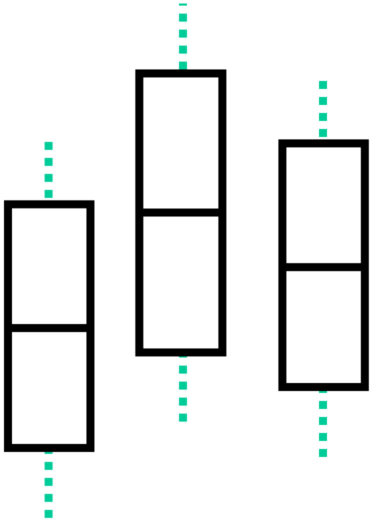
Creating Ideas**
Time Value Map
Process Redesign
IDEF0
Value Stream Mapping
SIPOC
Flow Process Chart
Process Mapping
Flowcharting
Service Blueprints
Designing & Analyzing Processes

- ANOVA

- Analysis of Variance.
- Used to determine whether the mean responses for two or more groups differ.
- We can use ANOVA to compare the means of three or more population.
- If we are only comparing two means, then ANOVA will give the same results as the 2-samples t-test.
- The Math is different, but the approach and interpretation of p-values is the same.



- ANOVA

- We must be clear of the hypotheses before applying the technique.
- A hypothesis test used to determine whether **two or more sample means are significantly different** by comparing the variances between groups.

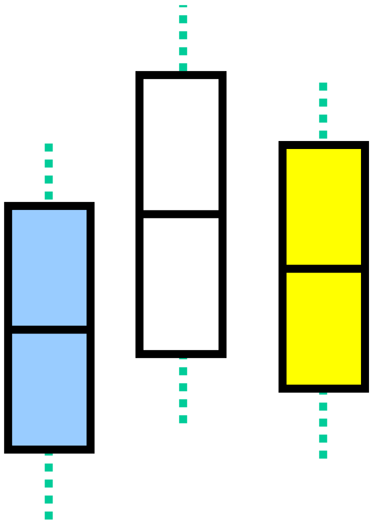
The Null Hypothesis → The sample means are all the same.
The Alternative Hypothesis → They are not all the same (at least one of them differs significantly from the others).

- Be careful how you phrase: "There is a difference" not "They are all different".

- ANOVA

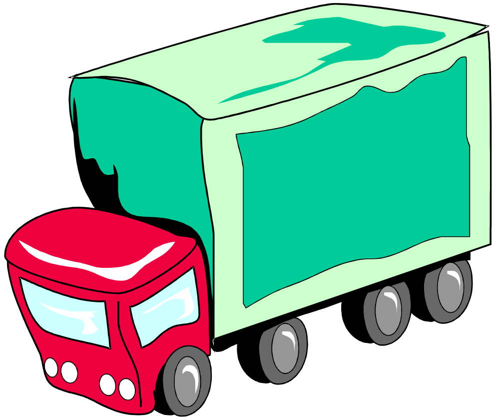
Some important terms used in ANOVA:

- **Factor:**
  - The explanatory variable in the study.
  - The factor is categorical (the data classify people, objects or events).
- **Levels:**
  - The groups or categories that comprise a factor.
- **Response:**
  - A variable (continuous) being measured in the study.



- ANOVA

Example:

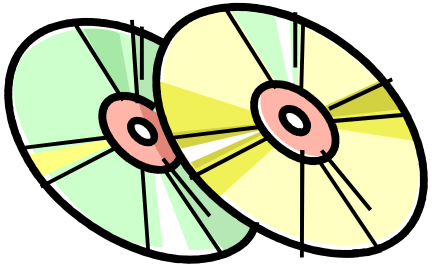


- If we select the supplier to be the **factor**, each supplier represent a **level** (the group within a factor).
- In **one-way ANOVA**, there is only one factor.
- ANOVA is used to compare the means of the factor levels to determine whether the levels differ.
- Where is the **response**?

- ANOVA

Example:

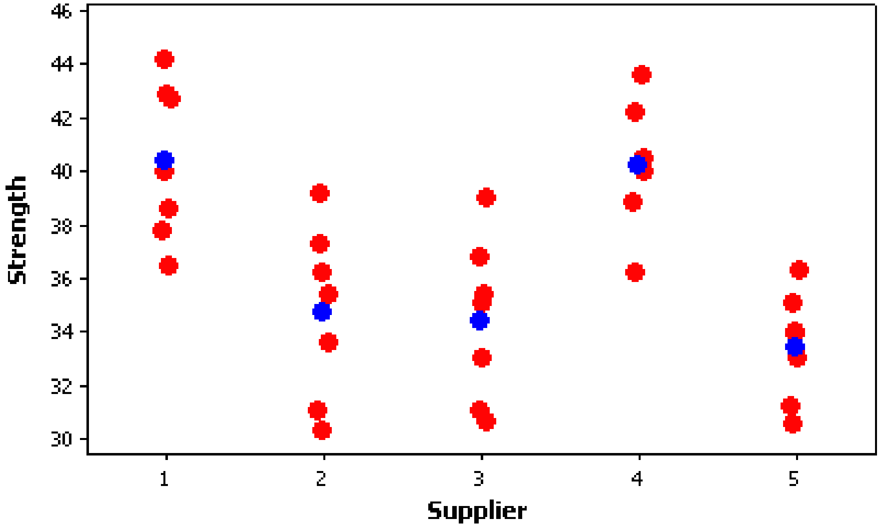
- If a company wants to purchase one of three expensive software packages:



- The software would be the **factor** because it is our explanatory variable.
- The three software packages are the **levels** that comprise the factor.
- The amount of time it takes to fill out a report would be the **response** (because it is the particular variable being measured).

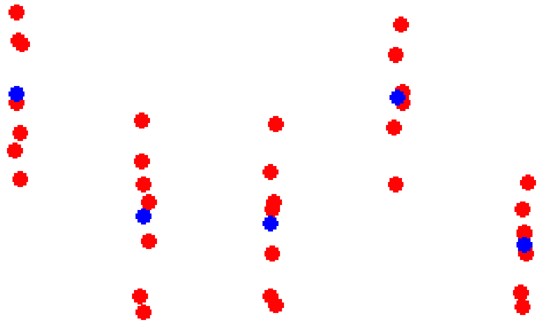
- ANOVA

- In ANOVA, it is useful to graph the data.
- We can examine the factor level means and look at the variation **within** each group and **between** all groups.
- However, the graph will give no idea if the differences between the means are statistically significant.



- ANOVA

- **Within-group variation** is the variability in measurements within individual groups.
- **Between-group variation** is the variability in measurements between all groups.
- We compare between-group variation to within-group variation to determine whether real differences between groups exist.
- If the between-group variation is large relative to the within-group variation, evidence suggests that the population means are not the same, and vice versa.



- ANOVA

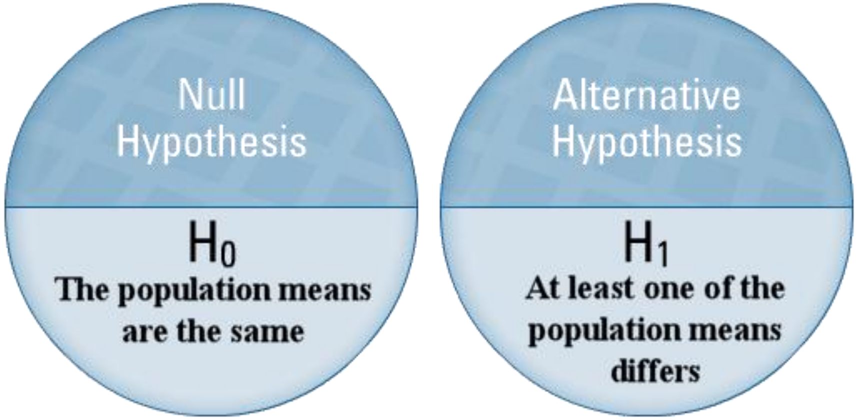
- A **test** to be conducted to decide whether differences between group means are real or simply random error.
- We can compare between and within group variations using **F-statistic ratio**.

$$\text{F-statistic} = \frac{\text{Between-group variation}}{\text{Within-group variation}}$$

- **When F is large →** between-group variation is larger than within group variation, which indicates a real difference between group means.
- **When F is small →** little or no evidence of a significant difference between group means.

- ANOVA

- When comparing the means of two population, we can use either the 2-sample t-test or **one-way ANOVA**.
- We must first define the null and alternative hypotheses.
- We need to use the **p-value** from the ANOVA output to determine whether we should reject or fail to reject the null hypothesis.



- ANOVA

Example:

- An automobile company uses nylon from five different suppliers to manufacture automobile safety belts.
- Suppose after establishing the hypothesis & collecting random samples, the results are:

One-way ANOVA: Strength versus Supplier

Source	DF	SS	MS	F	P
Supplier	4	322.63	80.66	10.50	0.000
Error	30	230.44	7.68		
Total	34	553.06			

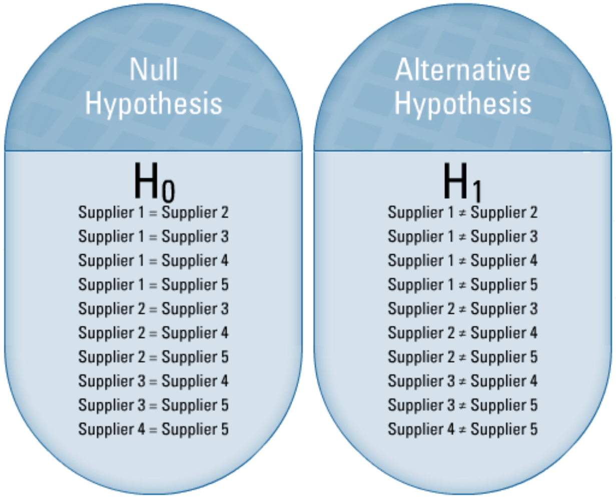
- As **p-value < 0.05**, we will reject the null hypotheses.
- The fiber strength for at least one supplier is different from the others.

- ANOVA

- In the previous example, we need to know which suppliers produce the strongest fiber.
- For this purpose, we can use multiple comparisons.
- The **multiple comparisons** are the simultaneous testing of multiple hypotheses.
- We will use a method called **Tukey's test** multiple comparisons, which checks for differences in pairs of group means.

- ANOVA

- Each individual comparison is like a **2-sample t-test**.
- For each comparison, we will examine the **confidence interval** to determine whether there is a significant difference between the groups.



- Each confidence interval provides a range of likely values for the difference between the two population means.
- If the confidence interval does not contain the value **zero**, then we reject the null hypothesis and conclude that the two group are different.

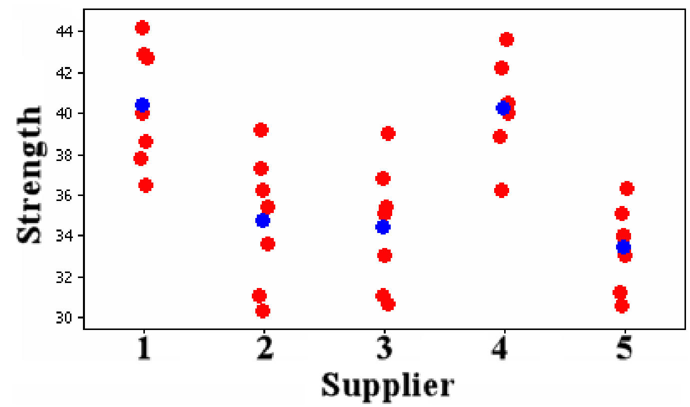
- ANOVA

- Here are the results of all the comparisons.
- For example, we will reject the null hypothesis for supplier 2 and 4.
- Therefore, there strength are different.

```
Supplier = 1 subtracted from:

Supplier     Lower   Center    Upper   ---------+---------+---------+---------+
2           -9.966   -5.671   -1.377     (-------*------)
3          -10.266   -5.971   -1.677     (------*------)
4           -4.423   -0.129    4.166               (------*------)
5          -11.252   -6.957   -2.662   (------*-------)
                                       ---------+---------+---------+---------+
                                             -6.0       0.0       6.0      12.0

Supplier = 2 subtracted from:

Supplier    Lower   Center   Upper   ---------+---------+---------+---------+
3          -4.595   -0.300   3.995             (-------*------)
4           1.248    5.543   9.838                       (------*------)
5          -5.581   -1.286   3.009            (------*------)
                                     ---------+---------+---------+---------+
                                           -6.0       0.0       6.0      12.0

Supplier = 3 subtracted from:

Supplier    Lower   Center    Upper   ---------+---------+---------+---------+
4           1.548    5.843   10.138                       (------*------)
5          -5.281   -0.986    3.309             (------*-------)
                                      ---------+---------+---------+---------+
                                            -6.0       0.0       6.0      12.0

Supplier = 4 subtracted from:

Supplier     Lower   Center    Upper   ---------+---------+---------+---------+
5          -11.123   -6.829   -2.534   (-------*------)
                                       ---------+---------+---------+---------+
                                             -6.0       0.0       6.0      12.0
```

- ANOVA

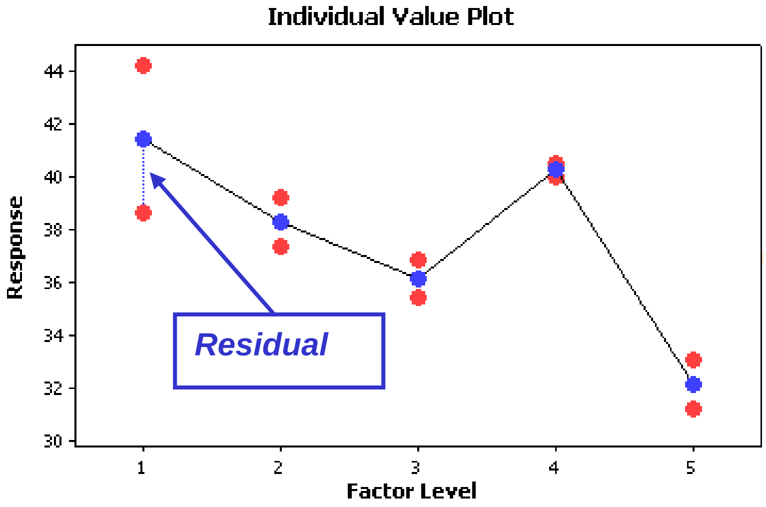
- **Question:** Which two suppliers have the higher mean strength measurements than the others?
- **Answer:** 1 and 4.
- **Question:** Is there a statistical difference between suppliers 1 and 4? Or which one is the absolute strongest?
- **Answer:** No, supplier 4 contains the value zero when comparing to supplier 1.




```
Supplier = 1 subtracted from:

Supplier    Lower   Center    Upper
2          -9.966   -5.671   -1.377
3         -10.266   -5.971   -1.677
4          -4.423   -0.129    4.166
5         -11.252   -6.957   -2.662


          ---------+---------+---------+---------+
  2        (-------*------)
  3        (------*-------)
  4                  (------*------)
  5      (------*-------)
          ---------+---------+---------+---------+
                -6.0       0.0       6.0      12.0
```

- ANOVA

- The **residuals** estimates the error in ANOVA.
- They are calculated by subtracting the observed value from the fitted value (the group mean if the sample size is the same for each group).
- We can examine the plots of the residuals to check the ANOVA assumptions.



- Errors in ANOVA (residuals) should be random independent, normally distributed and have constant variance across all factor levels.

- ANOVA

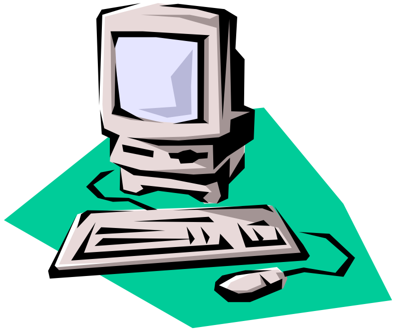
- When we have two factors, we can use two-way ANOVA to investigate differences among group means.
- In two-way ANOVA, we use the one-way ANOVA terms (factor, levels and response).

- **New terms:**
  - **Main effect:** The influence of a single factor on a response.
  - **Interaction:** An interaction between factors is present when the mean response for the levels of one factor depends on the level of the second factor.

- ANOVA

Example:

- An IT Consultancy employs a variety of software developers to provide custom software solutions.
- It has programmers, testers and system administrators.
- Company's training programs are classroom teaching, instructional videotapes, and one-on-one training.
- The manager wants to determine **how to best leverage the company's training programs.**
- It can save the company money in the long run if the right employees are trained in the best possible.



- ANOVA

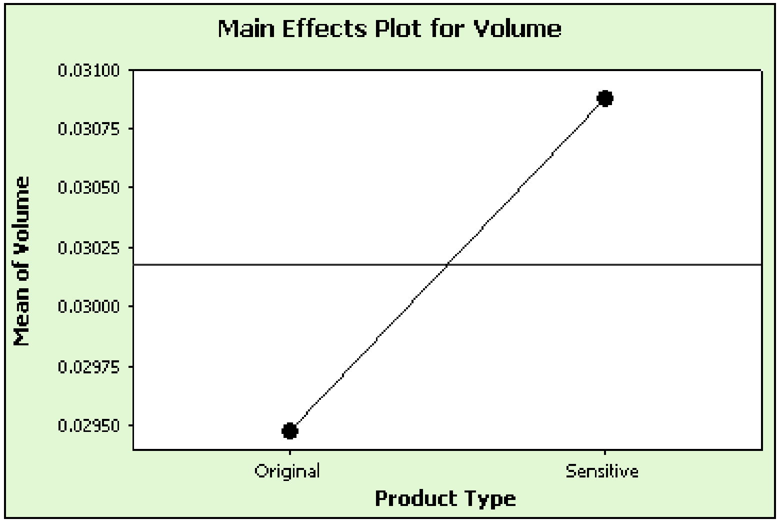
Example:

- **Factors:**
  - Job type & method of instruction.

- **Levels:**
  - **Job type:** programmers, testers & system administrators.
  - **Method of instruction:** classroom teaching, instructional videotapes, and one-on-one training.
- **Response:**
  - Impact on employees **(effectiveness of training via test)**.
- **Main effect and Interaction:**
  - If a particular job type achieves higher scores from using a given method of instruction, is this an **interaction** or a **main effect**?

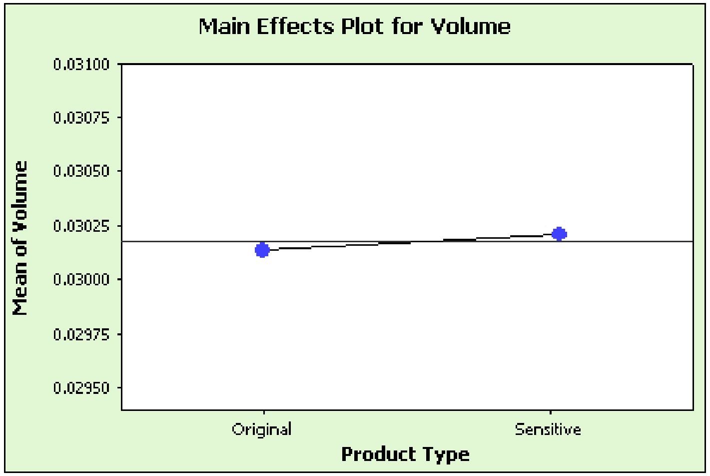
- ANOVA

- How can we get a better idea of how factors might be affecting the response?
- The main effects plot can show whether each factor **individually** influences the response.
- First we'll calculate the mean for each level of the two factors.
- Then we plot these values on the graph and draw a line to connect the points.
- Finally, we add a reference line at the overall mean of the data.



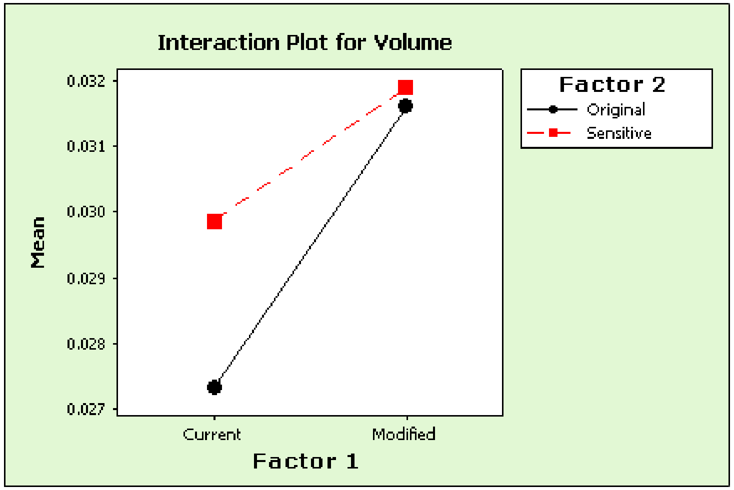
- ANOVA

- If the line connecting the points is horizontal, this indicates that no main effect is present.
- If the output line is not horizontal, the main effect is present.
- The greater the slope of the line, the stronger the effect.
- Remember to include measurements of all factor levels for the other factor.



- ANOVA

- To show how the effect of one factor interacts with the effect of another, we'll use the interaction plot.
- We need to calculate the mean of each combination of levels for the two factors.
- Then we'll connect each pair of points with a line (in different color).
- When an interaction exists, the connecting lines on the interaction plot are not parallel. They intersect or almost intersect (as shown here).



- ANOVA

2 Ways ANOVA Approach:

- Establish the hypothesis.
  - For each factor.
  - For the interaction between the factors.

The Null Hypothesis → Either no main effect is present or no interaction effect is present.
The Alternative Hypothesis → Either a main effect is present or an interaction effect is present.

- Collect random samples.
- Graph the data (main effects and interaction plots).
- Conduct the 2 Ways ANOVA and interpret the results.

Example:

- Let's get back to the IT consultancy study.
- The null and the alternative hypothesis:

H0: Mean scores on the test are the same of each training method.
H1: Mean scores on the test are different for at least one of the training methods.
H0: Mean scores on the test are the same of all job types.
H1: Mean scores on the test are different for at least 1 of the job types.
H0: The Change in the mean response across levels of training methods does not depend on job type.
H1: The Change in the mean response across levels of training methods does depend on job type.

- ANOVA

Example:

- In two-way ANOVA, we first need to test whether a significant interaction effect is present.
- We will need to consider the **F-statistic** and the **p-value** to determine whether the interaction statistically significant.
- If the interaction is significant, it does not make sense to look at the main effect for each factor individually.
- Like in one-way ANOVA, we can examine the plots of the residuals to check the ANOVA assumptions.

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Factor 1	1	0.0000795	0.0000795	0.0000795	6.36	0.013
Factor 2	1	0.0004007	0.0004007	0.0004007	32.07	0.000
Factor 1 * Factor 2	1	0.0000504	0.0000504	0.0000504	4.03	0.046
Error	156	0.0019492	0.0019492	0.0000125		
Total	159	0.0024798				