Cumulative Sum and Exponentially Weighted Moving Average Control Charts

Alternative Control Charts to Shewhart Charts

- Control charts presented so far have been Shewhart control charts.
  - Uses information about the process contained in the last plotted point.
  - Shewhart charts are relatively insensitive to small process shifts, e.g. shifts $< 1.5\sigma$.
- Two alternative control charts for process monitoring.
  - CUSUM – Cumulative-sum control chart.
  - EWMA – Exponentially Weighted Moving Average control chart.

Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	−0.55	−0.55
2	7.99	−2.01	−2.56
3	9.29	−0.71	−3.27
4	11.66	1.66	−1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	−1.96	−1.23
8	11.46	1.46	0.23
9	9.20	−0.80	−0.57
10	10.34	0.34	−0.23
11	9.03	−0.97	−1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	−0.60	0.18
15	10.08	0.08	0.26
16	9.37	−0.63	−0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	−1.48	−0.92
20	10.84	0.84	−0.08
21	10.90	0.90	0.82
22	9.33	−0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

$$\text{UCL} = 13$$
$$\text{Center line} = 10$$
$$\text{LCL} = 7$$

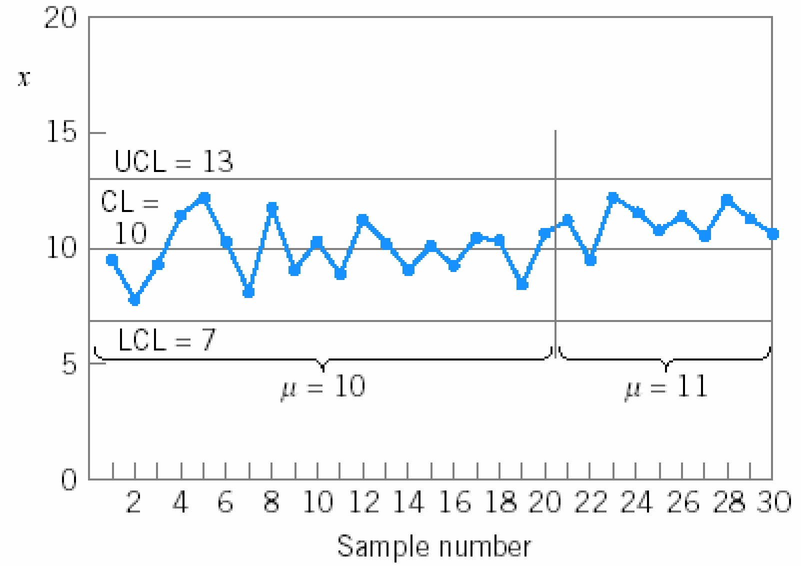


Figure 8-1 A Shewhart control chart for the data in Table 8-1.

Alternative Control Charts to Shewhart Charts

- Two alternative control charts for process monitoring.
  - CUSUM – Cumulative-sum control chart.
  - EWMA – Exponentially Weighted Moving Average control chart.

CUSUM Control Chart

- More sensitive to small process shifts than Shewart control charts.
- CUSUM can be used to monitor
  - process mean
  - defectives
  - defects
  - variance
- CUSUM can have sample size $n \geq 1$

CUSUM Control Chart

- Incorporates all the information in the sequence of sample values by
  - Monitoring the cumulative sums of the deviations of the sample values from a target value, μ_0

$$C_i = \sum_{j=1}^{i} (\bar{x}_j - \mu_0)$$

Example

- C_i – CUSUM sample statistic $C_i = \sum_{j=1}^{i} (x_j - \mu_0)$ since n=1.
- Target μ_0 = 10

$$C_i = \sum_{j=1}^{i} (x_j - 10) = (x_j - 10) + \sum_{j=1}^{i-1} (x_j - 10) = (x_j - 10) + C_{i-1}$$

Example

Obs i	x_i	$(x_i - \mu_0)$	$(x_i - 10) + C_{i-1}$
1	9.45	-0.55	-0.55
2	7.99	-2.01	-0.55 – 2.01 = -2.56
3	9.29	-0.71	-2.56 – 0.71 = -3.27
4	11.66	1.66	-3.27 + 1.66 = -1.61
5	12.16	2.16	-1.61 + 2.16 = 0.55

- If the process remains in-control, C_i remains near 0.

Table 8-1 Data for the Cusum Example

Sample, i	(a) x_i	(b) $x_i - 10$	(c) $C_i = (x_i - 10) + C_{i-1}$
1	9.45	–0.55	–0.55
2	7.99	–2.01	–2.56
3	9.29	–0.71	–3.27
4	11.66	1.66	–1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	–1.96	–1.23
8	11.46	1.46	0.23
9	9.20	–0.80	–0.57
10	10.34	0.34	–0.23
11	9.03	–0.97	–1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	–0.60	0.18
15	10.08	0.08	0.26
16	9.37	–0.63	–0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	–1.48	–0.92
20	10.84	0.84	–0.08
21	10.90	0.90	0.82
22	9.33	–0.67	0.15
23	12.29	2.29	2.44
24	11.50	1.50	3.94
25	10.60	0.60	4.54
26	11.08	1.08	5.62
27	10.38	0.38	6.00
28	11.62	1.62	7.62
29	11.31	1.31	8.93
30	10.52	0.52	9.45

CUSUM Plot

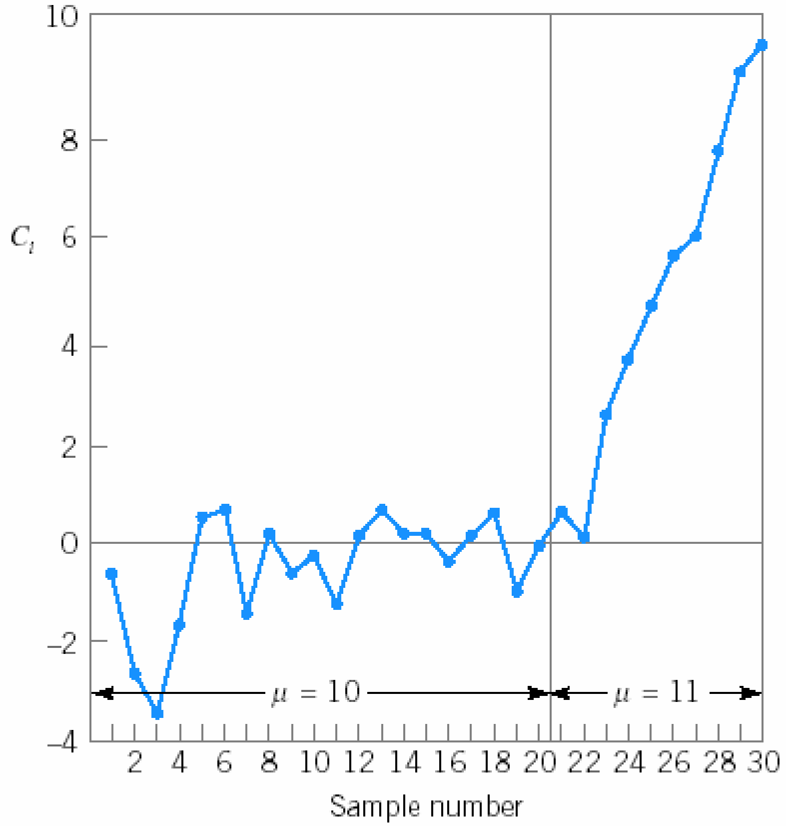


Figure 8-2 Plot of the cumulative sum from column (c) of Table 8-1.

Not a control chart- No control limits have been established.

Tabular CUSUM Control Chart

- $x_i \sim N(\mu_0, \sigma)$ - quality characteristic

- The tabular CUSUM works by compiling the statistics:
  - C^+ = accumulated deviations above μ_0
  - C^- = accumulated deviations below μ_0
  - Record following values in a table (for n=1):

$$C_i^+ = \max\left[0, x_i - (\mu_0 + K) + C_{i-1}^+\right]$$

$$C_i^- = \max\left[0, (\mu_0 - K) - x_i + C_{i-1}^-\right]$$

where starting values are

$$C_0^+ = C_0^- = 0$$

Tabular CUSUM Control Chart

- Let μ_1 = out-of-control value then

$$K = \frac{|\mu_1 - \mu_0|}{2}$$

  - *K* is called the reference value, and it is halfway between the target $\mu_{0,}$ and the out-of-control value.

- With the μ_1- μ_0 difference expressed in standard deviation units,

$$\mu_1 = \mu_0 + \delta\sigma \Rightarrow \delta\sigma = |\mu_1 - \mu_0|$$

$$K = \frac{\delta\sigma}{2}$$

Tabular CUSUM Control Chart

- C_i^+ and C_i^- accumulate only deviations from the target that are greater than *K*.
- If the deviation is less than K, C_i^+ and C_i^- are zero.

EXAMPLE 8-1

A Tabular Cusum

We will demonstrate the calculations for the tabular cusum by using the data from Table 8-1. Recall that the target value is $\mu_0 = 10$, the subgroup size is $n = 1$, the process standard deviation is $\sigma = 1$, and suppose that the magnitude of the shift we are interested in detecting is $1.0\sigma = 1.0(1.0) = 1.0$. Therefore, the out-of-control value of the process mean is $\mu_1 = 10 + 1 = 11$. We will use a tabular cusum with $K = \frac{1}{2}$ (because the shift size is 1.0σ and $\sigma = 1$) and $H = 5$ (because the recommended value of the decision interval is $H = 5\sigma = 5(1) = 5$).

Table 8-2 presents the tabular cusum scheme. To illustrate the calculations, consider period 1. The equations for C_i^+ and C_i^- are

$$C_1^+ = \max\left[0, x_1 - 10.5 + C_0^+\right]$$

and

$$C_1^- = \max\left[0, 9.5 - x_1 + C_0^-\right]$$

since $K = 0.5$ and $\mu_0 = 10$. Now $x_1 = 9.45$, so since $C_0^+ = C_0^- = 0$,

$$C_1^+ = \max[0, 9.45 - 10.5 + 0] = 0$$

and

$$C_1^- = \max[0, 9.5 - 9.45 + 0] = 0.05$$

$\mu_0 + K$

$\mu_0 - K$

$\mu_0 = 10$

$K = 1/2$

$x_1 = 9.45$

$x_2 = 7.99$

$x_3 = 9.29$

For period 2, we would use

$$C_2^+ = \max\left[0, x_2 - 10.5 + C_1^+\right]$$
$$= \max\left[0, x_2 - 10.5 + 0\right]$$

and

$$C_2^- = \max\left[0, 9.5 - x_2 + C_1^-\right]$$
$$= \max\left[0, 9.5 - x_2 + 0.05\right]$$

Since $x_2 = 7.99$, we obtain

$$C_2^+ = \max[0, 7.99 - 10.5 + 0] = 0$$

and

$$C_2^- = \max[0, 9.5 - 7.99 + 0.05] = 1.56$$

Table 8-2 The Tabular Cusum for Example 8-1

		(a)			(b)		
Period i	x_i	$x_i - 10.5$	C_i^+	N^+	$9.5 - x_i$	C_i^-	N^-
1	9.45	−1.05	0	0	0.05	0.05	1
2	7.99	−2.51	0	0	1.51	1.56	2
3	9.29	−1.21	0	0	0.21	1.77	3
4	11.66	1.16	1.16	1	−2.16	0	0
5	12.16	1.66	2.82	2	−2.66	0	0
6	10.18	−0.32	2.50	3	−0.68	0	0
7	8.04	−2.46	0.04	4	1.46	1.46	1
8	11.46	0.96	1.00	5	−1.96	0	0
9	9.20	−1.3	0	0	0.30	0.30	1
10	10.34	−0.16	0	0	−0.84	0	0
11	9.03	−1.47	0	0	0.47	0.47	1
12	11.47	0.97	0.97	1	−1.97	0	0
13	10.51	0.01	0.98	2	−1.01	0	0
14	9.40	−1.10	0	0	0.10	0.10	1
15	10.08	−0.42	0	0	−0.58	0	0
16	9.37	−1.13	0	0	0.13	0.13	1
17	10.62	0.12	0.12	1	−1.12	0	0
18	10.31	−0.19	0	0	−0.81	0	0
19	8.52	−1.98	0	0	0.98	0.98	1
20	10.84	0.34	0.34	1	−1.34	0	0
21	10.90	0.40	0.74	2	−1.40	0	0
22	9.33	−1.17	0	0	0.17	0.17	1
23	12.29	1.79	1.79	1	−2.79	0	0
24	11.50	1.00	2.79	2	−2.00	0	0
25	10.60	0.10	2.89	3	−1.10	0	0
26	11.08	0.58	3.47	4	−1.58	0	0
27	10.38	−0.12	3.35	5	−0.88	0	0
28	11.62	1.12	4.47	6	−2.12	0	0
29	11.31	0.81	5.28	7	−1.81	0	0
30	10.52	0.02	5.30	8	−1.02	0	0

Both C_i^+ and C_i^- values are compared to $H=5$ to determine control status

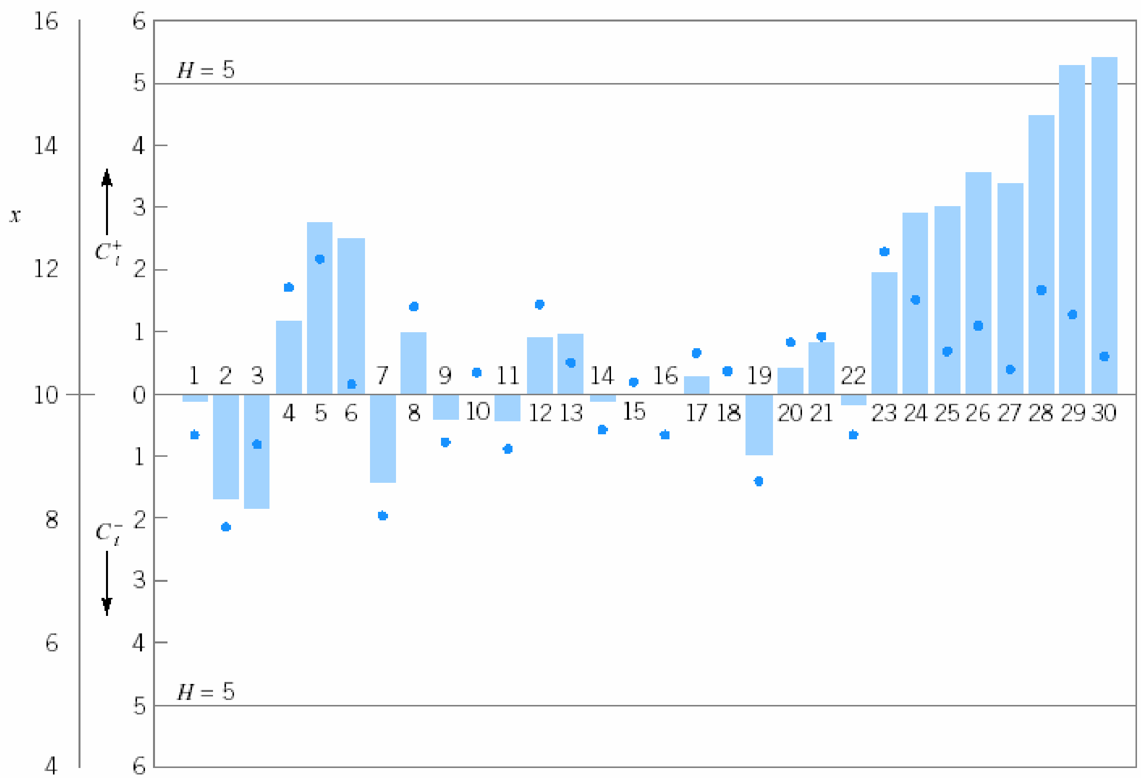
C_i^+ and C_i^- will both be zero if x_i is within $N_0 \pm K$ and $C_{i-1}^+, C_{i-1}^- = 0$

Control Limits

- *H* – Called the <u>decision interval.</u>
- If C_i^+ or C_i^- exceed the decision interval, *H*, the process is considered out-of-control.
- Rule of thumb value for *H*
  - Choose *H* to be five times the process standard deviation, i.e., $H = 5\sigma$.
- Counters N^+ and N^- record the number consecutive periods the CUSUM C_i^+ and C_i^- are above zero.
  - The counters can be used to indicate when a process shift most likely occurred.

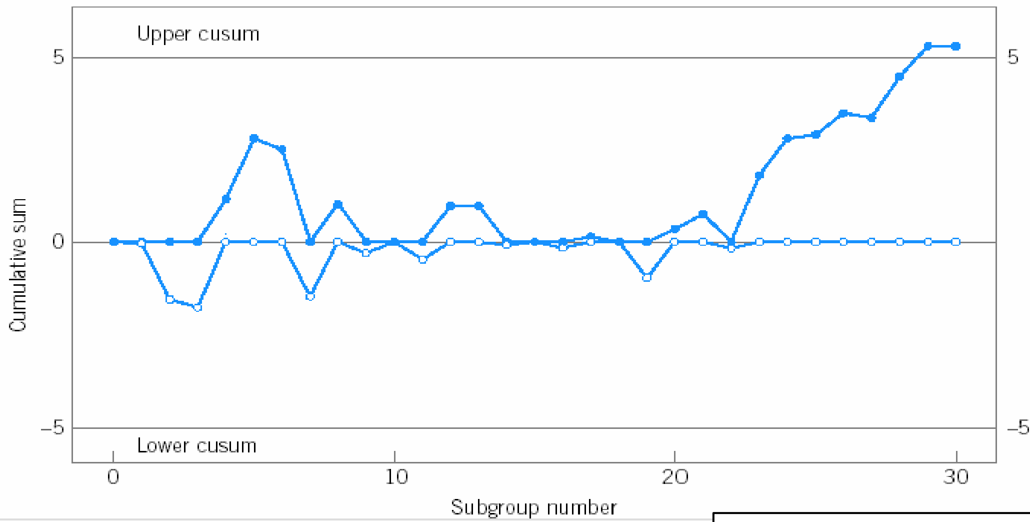
CUSUM Status Chart

- *A control chart showing the control limits (defined by H) and* C_i^+ C_i^-



CUSUM Status Chart

- *A control chart showing the control limits (defined by H) and* C_i^+ C_i^-



$$C_i^- = \min\left(0, x_i - \mu_0 + k + C_{i-1}^-\right)$$

negative of the formulas presented.

MINITAB calculates the lower cusum this way

Intuition of the CUSUM Chart

1. A weighted sum of all past observations (deviations from a constant) where the weighting is random.
2. Each time C+ or C- is zero, the value monitored “restarts”.

Table 8-2 The Tabular Cusum for Example 8-1

		(a)			(b)		
Period i	x_i	$x_i - 10.5$	C_i^+	N^+	$9.5 - x_i$	C_i^-	N^-
1	9.45	−1.05	0	0	0.05	0.05	1
2	7.99	−2.51	0	0	1.51	1.56	2
3	9.29	−1.21	0	0	0.21	1.77	3
4	11.66	1.16	1.16	1	−2.16	0	0
5	12.16	1.66	2.82	2	−2.66	0	0
6	10.18	−0.32	2.50	3	−0.68	0	0
7	8.04	−2.46	0.04	4	1.46	1.46	1
8	11.46	0.96	1.00	5	−1.96	0	0
9	9.20	−1.3	0	0	0.30	0.30	1
10	10.34	−0.16	0	0	−0.84	0	0
11	9.03	−1.47	0	0	0.47	0.47	1
12	11.47	0.97	0.97	1	−0.47	0	0
13	10.51	0.01	0.98	2	−1.01	0	0
14	9.40	−1.10	0	0	0.10	0.10	1
15	10.08	−0.42	0	0	−0.58	0	0
16	9.37	−1.13	0	0	0.13	0.13	1
17	10.62	0.12	0.12	1	−1.12	0	0
18	10.31	−0.19	0	0	−0.81	0	0
19	8.52	−1.98	0	0	0.98	0.98	1
20	10.84	0.34	0.34	1	−1.34	0	0
21	10.90	0.40	0.74	2	−1.40	0	0
22	9.33	−1.17	0	0	0.17	0.17	1
23	12.29	1.79	1.79	1	−2.79	0	0
24	11.50	1.00	2.79	2	−2.00	0	0
25	10.60	0.10	2.89	3	−1.10	0	0
26	11.08	0.58	3.47	4	−1.58	0	0
27	10.38	−0.12	3.35	5	−0.88	0	0
28	11.62	1.12	4.47	6	−2.12	0	0
29	11.31	0.81	5.28	7	−1.81	0	0
30	10.52	0.02	5.30	8	−1.02	0	0

Example – Problem 8-1

- The target value of molecular weight is 1050 and the process standard deviation is thought to be about 25.
  - Set up a tabular CUSUM for the mean of this process. Design it so that to detect shifts of one standard deviation from the target.
  - Is the estimate of σ a good estimate?

$\mu_0 = 1050$ $K = \frac{1}{2}\sigma = \frac{1}{2} * 25 = 12.5$ $H = 50$

$\mu_0 + K = 1062.5$

$\mu_0 - K = 1037.5$

$H = 5 * 25 = 125$

Example – Problem 8-1

Obser. #	Weight	$x_i - 1062.5$	C_i^+	N^+	$1037.5 - x_i$	C_i^-	N^-
1	1045	-17.5	0	0	-7.5	0	0
2	1055	-7.5	0	0	-17.5	0	0
3	1037	-25.5	0	0	0.5	0.5	1
4	1064	1.5	1.5	1	-26.5	0	0
5	1095	32.5	34	2	-57.5	0	0
6	1008	-54.5	0	0			
7	1050						
8	1087						
9	1125						
10	1146						
11	1139						
12	1169						
13	1151						
14	1128						
15	1238						
16	1125						
17	1163						
18	1188						
19	1146						
20	1167						

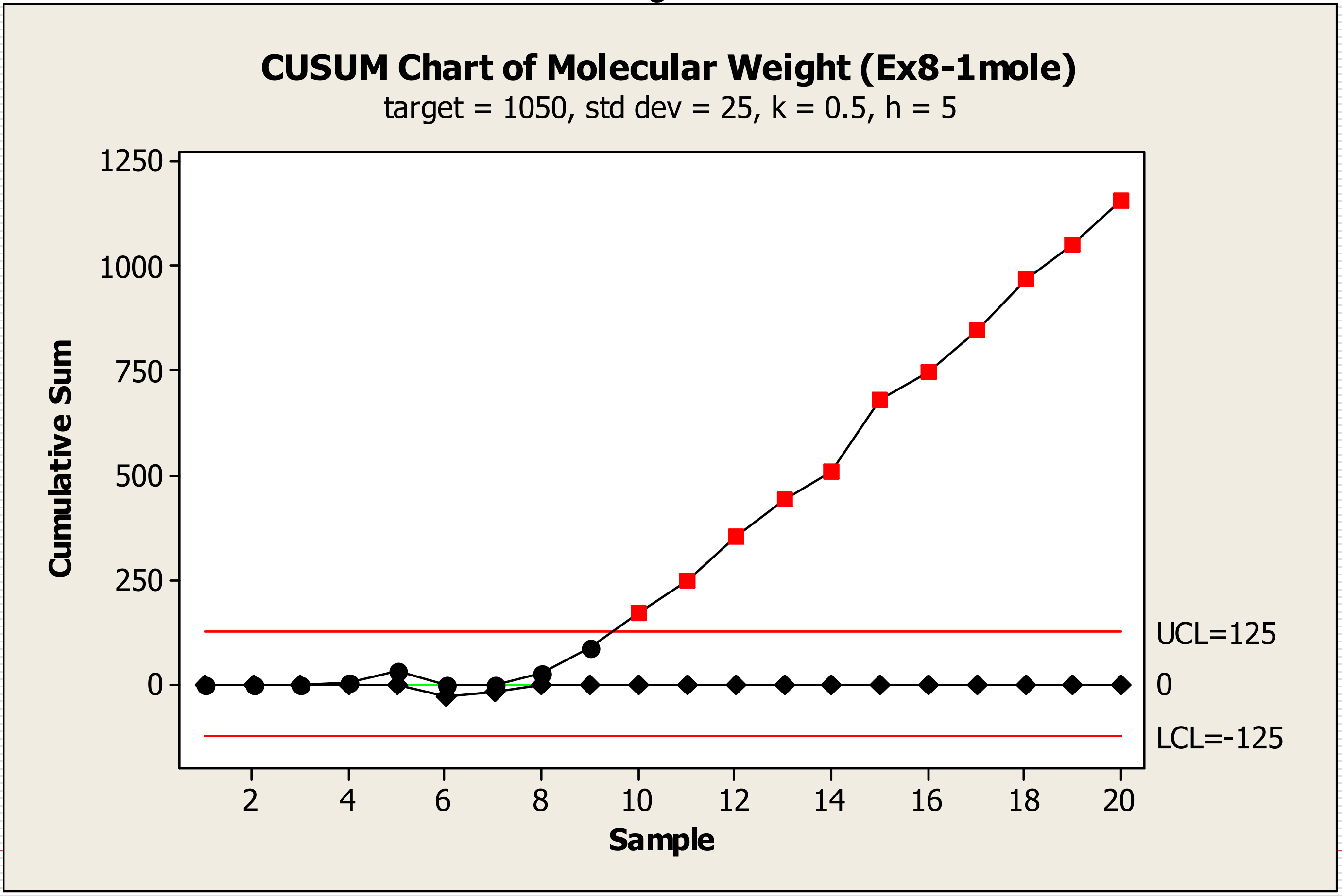
$n = 1$

Example – Problem 8-1

Obser. #	Weight	xi-1062.5	C+	N+	xi-1037.5	C-	N-
1	1045	-17.5	0	0	-7.5	0	0
2	1055	-7.5	0	0	-17.5	0	0
3	1037	-25.5	0	0	0.5	0.5	1
4	1064	1.5	1.5	1	-26.5	0	0
5	1095	32.5	34	2	-57.5	0	0
6	1008	-54.5	0	0	29.5	29.5	1
7	1050	-12.5	0	0	-12.5	17	2
8	1087	24.5	24.5	1	-49.5	0	0
9	1125	62.5	87	2	-87.5	0	0
10	1146	83.5	170.5	3	-108.5	0	0
11	1139	76.5	247	4	-101.5	0	0
12	1169	106.5	353.5	5	-131.5	0	0
13	1151	88.5	442	6	-113.5	0	0
14	1128	65.5	507.5	7	-90.5	0	0
15	1238	175.5	683	8	-200.5	0	0
16	1125	62.5	745.5	9	-87.5	0	0
17	1163	100.5	846	10	-125.5	0	0
18	1188	125.5	971.5	11	-150.5	0	0
19	1146	83.5	1055	12	-108.5	0	0
20	1167	104.5	1159.5	13	-129.5	0	0

Example – Problem 8-1

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Example – Problem 8-1

Obser. #	Weight	MR
1	1045	
2	1055	10
3	1037	18
4	1064	27
5	1095	31
6	1008	87
7	1050	42
8	1087	37
9	1125	38
10	1146	21
11	1139	7
12	1169	30
13	1151	18
14	1128	23
15	1238	110
16	1125	113
17	1163	38
18	1188	25
19	1146	42
20	1167	21

avg. MR 38.84211

$$\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{38.842}{1.128} = 34.43$$

for $n=2$

$\Rightarrow$ 25 for σ is too low

Estimate of New Shifted Process Mean

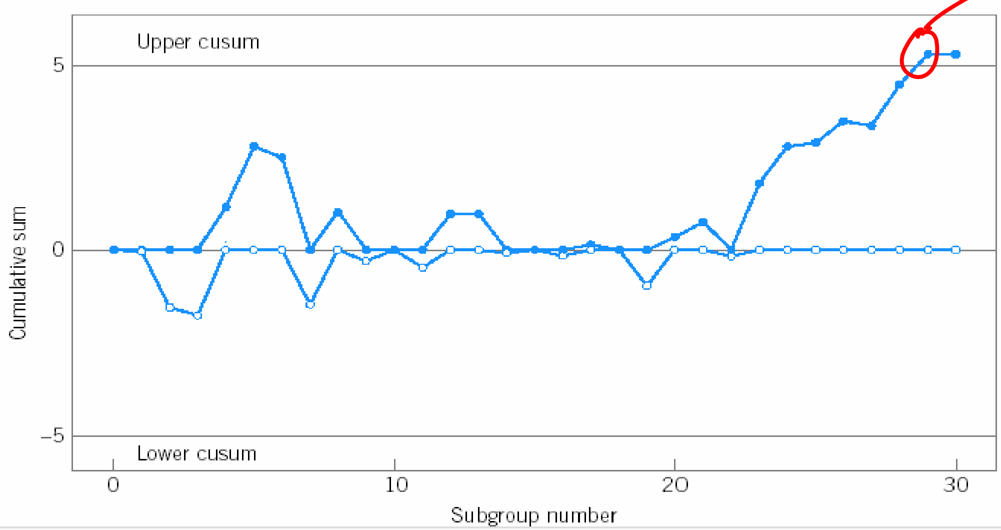
$$\hat{\mu} = \begin{cases} \mu_0 + K + \dfrac{C_i^+}{N^+}, & \text{if } C_i^+ > H \\ \mu_0 - K - \dfrac{C_i^-}{N^-}, & \text{if } C_i^- > H \end{cases}$$

- Use this estimate to bring process back to the target value μ_0
- In the first example H=5 and $C_{29}^+ = 5.28$
- New process average estimate is

$$\hat{\mu} = \mu_0 + K + \frac{C_{29}^+}{N^+} = 10 + 0.5 + \frac{5.28}{7} = 11.25$$

First Example

- *A control chart showing the control limits (defined by H) and* C_i^+ C_i^-



$C_{29}^+ = 5.28$

$\hat{\mu} = 10 + 0.5 + \frac{5.28}{7} = 11.25$

represents ?

An Average of the deviations over $\mu_0 + K$

Recommendations for CUSUM Charts

- Let $H = h\sigma$ and $K = k\sigma$ where σ is the process standard deviation.
- Using h = 4 or h =5 and k = 1/2 provides a CUSUM with good ARL properties.
- For n>1 replace x_i with $x_i\ bar$ and σ with $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

Standardized CUSUM

- CUSUM charts can be constructed for standardized observations.

$$y_i = \frac{x_i - \mu_0}{\sigma}$$

what is the mean? zero
what is the std dev? one

$$C_i^+ = \max[0, y_i - k + C_{i-1}^+]$$

$$C_i^- = \max[0, -y_i - k + C_{i-1}^-]$$

- Can use k=1/2, h= 4 or 5 for all charts.

Example – Problem 8-2

- The target value of molecular weight is 1050 and the process standard deviation is thought to be about 25.
  - Set up a standardized CUSUM for the mean of this process. Design it so that to detect shifts of one standard deviation from the target.

$\mu_0 = 1050$ $\sigma = 25$

$\frac{x_i - 1050}{25}$

Example – Problem 8-2

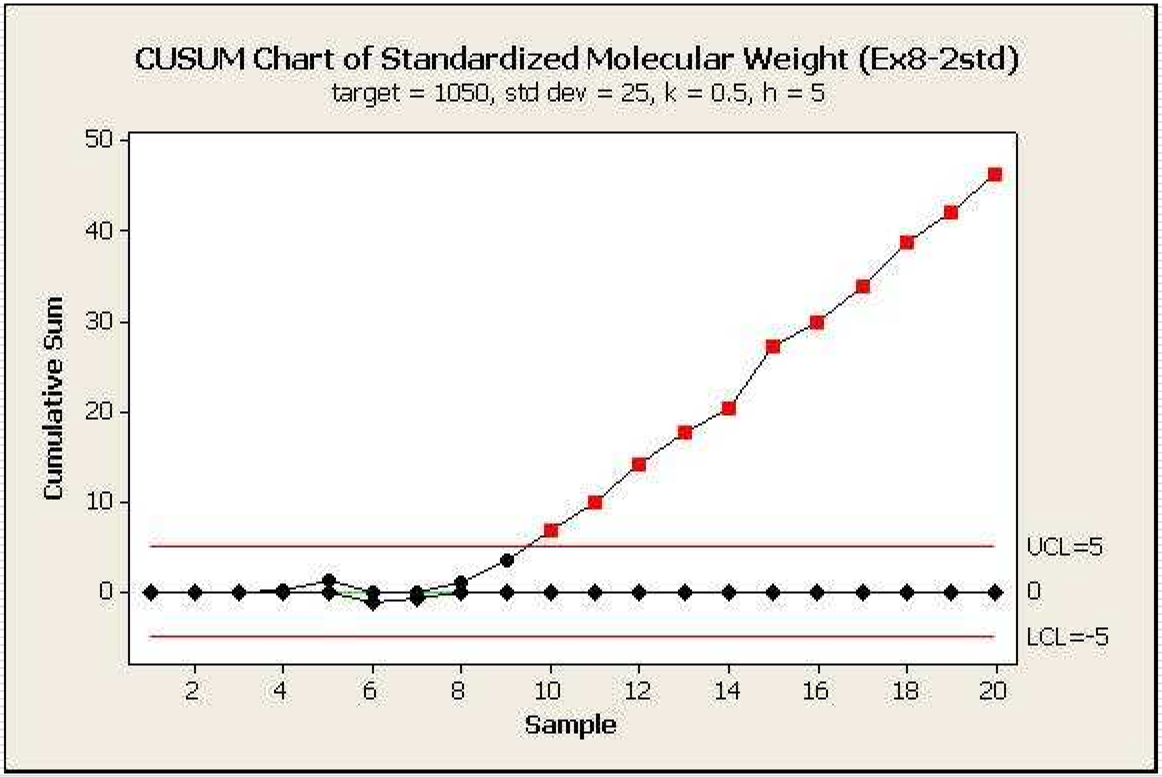
Obser. #	Weight	Std wt.	Std wt − 0.5	−Std wt − 0.5	C^+	N^+	
1	1045	−0.2	−0.7	−0.3	0	0	
2	1055	0.2	−0.3	−0.7	0	0	
3	1037	−0.52	−1.02	0.02	0	0	
4	1064	0.56	0.06	−1.06	0.06	1	
5	1095						
6	1008						
7	1050						
8	1087						
9	1125						
10	1146						
11	1139						
12	1169						
13	1151						
14	1128						
15	1238						
16	1125						
17	1163						
18	1188						
19	1146						
20	1167						

Example – Problem 8-2

Obser. #	Wt.	Std. Wt.	C+	N+	C-	N-
1	1045	-0.2	0	0	0	0
2	1055	0.2	0	0	0	0
3	1037	-0.52	0	0	0.02	1
4	1064	0.56	0.06	1	0	0
5	1095	1.8	1.36	2	0	0
6	1008	-1.68	0	0	1.18	1
7	1050	0	0	0	0.68	2
8	1087	1.48	0.98	1	0	0
9	1125	3	3.48	2	0	0
10	1146	3.84	6.82	3	0	0
11	1139	3.56	9.88	4	0	0
12	1169	4.76	14.14	5	0	0
13	1151	4.04	17.68	6	0	0
14	1128	3.12	20.3	7	0	0
15	1238	7.52	27.32	8	0	0
16	1125	3	29.82	9	0	0
17	1163	4.52	33.84	10	0	0
18	1188	5.52	38.86	11	0	0
19	1146	3.84	42.2	12	0	0
20	1167	4.68	46.38	13	0	0

Example – Problem 8-2

Example – Problem 8-2



EWMA - Exponentially Weighted Moving Average Control Chart

- The EWMA control chart is good for detecting small shifts.
- EWMA can be used to monitor process mean or variance.

- The EWMA is
  - $z_i = \lambda x_i + (1- \lambda)z_{i-1}$

- Use estimate for $z_0 = \hat{\mu}$ or a target value $z_0 = \mu_0$
- λ is weighting factor, where $0 < \lambda < 1$.

Example

- A process has a mean that is estimated to be 14.31. The first three observations of the process quality measure are shown in the table.
- Compute the EWMA statistic, z_i, with weight $\lambda = 0.2$.

Obs *i*	x_i	EWMA $z_i = \lambda x_i + (1-\lambda) z_{i-1}$
1	14.56	$z_1 = (0.2)(14.56) + (0.8)(14.31)$ $= 14.36$
2	13.88	$z_2 = (0.2)(13.88) + (0.8)(14.36)$ $= 14.26$
3	13.98	$z_3 = (0.2)(13.98) + (0.8)(14.26)$ $= 14.21$

EWMA - Exponentially Weighted Moving Average Control Chart

- The EWMA z_i is a weighted average of all observations since

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} = \lambda x_i + (1-\lambda)[\lambda x_{i-1} + (1-\lambda) z_{i-2}]$$

$$= \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0$$

and the weights sum to one.

$$= \lambda \sum_{j=0}^{i-1} (1-\lambda)^j + (1-\lambda)^i = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)} \right] + (1-\lambda)^i = 1$$

Since $1 + r + r^2 + \cdots + r^n = \frac{(1+r^{n+1})}{1-r}$

EWMA Control Limits

- Use a target value μ_0 for the centerline (may be a desired target or estimated process mean).
- Computing σ_z. Since the x_i observations are assumed independent with variance σ^2:

$$\sigma_{z_i}^2 = \sum_{j=0}^{i-1} \lambda^2 (1-\lambda)^{2j} \sigma^2 = \lambda^2 \sigma^2 \sum_{j=0}^{i-1} [(1-\lambda)^2]^j = \lambda^2\sigma^2\left(\frac{1-(1-\lambda)^{2i}}{1-(1-\lambda)^2}\right) = \sigma^2\frac{[1-(1-\lambda)^{2i}]\lambda}{2-\lambda}$$

z_i — *variance of x_i*

- Can estimate σ_z from the data if needed.

$$\hat{\sigma}_z = \hat{\sigma}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$\hat{\sigma} = \overline{MR}/d_2$$

EWMA Control Limits

- Control Limits and centerline. σ_z

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}$$

$$\text{Centerline} = \mu_0$$

σ_z

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}$$

- Limits change as a function of i.
- Typical values for λ and L:
  - $0.05 \le \lambda \le 0.25$ and $2.6 \le L \le 3.054$

EWMA Control Limits

- Steady state control limits and centerline.

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$\text{Centerline} = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$[1-(1-\lambda)^{2i}]$ approaches 1 as i gets larger.

EXAMPLE 8-2

We will apply the EWMA control chart with $\lambda = 0.10$ and $L = 2.7$ to the data in Table 8-1. Recall that the target value of the mean is $\mu_0 = 10$ and the standard deviation is $\sigma = 1$. The calculations for the EWMA control chart are summarized in Table 8-9, and the control chart (from Minitab) is shown is Fig. 8-7.

To illustrate the calculations, consider the first observation, $x_1 = 9.45$. The first value of the EWMA is

$$\begin{aligned} z_1 &= \lambda x_1 + (1-\lambda) z_0 \\ &= 0.1(9.45) + 0.9(10) \\ &= 9.945 \end{aligned}$$

Therefore, $z_1 = 9.945$ is the first value plotted on the control chart in Fig. 8-7. The second value of the EWMA is

$x_2 = 7.99$

$$\begin{aligned} z_2 &= \lambda x_2 + (1-\lambda) z_1 \\ &= 0.1(7.99) + 0.9(9.945) \\ &= 9.7495 \end{aligned}$$

Period 1 Control Limits:

$\sigma = 1$

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 10 + 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]}$$

$$= 10.27$$

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 10 - 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}\left[1-(1-0.1)^{2(1)}\right]}$$

$$= 9.73$$

Steady-State Control Limits:

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$= 10 + 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}}$$

$$= 10.62$$

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$$= 10 - 2.7(1)\sqrt{\frac{0.1}{(2-0.1)}}$$

$$= 9.38$$

$\sigma/\sqrt{n}$

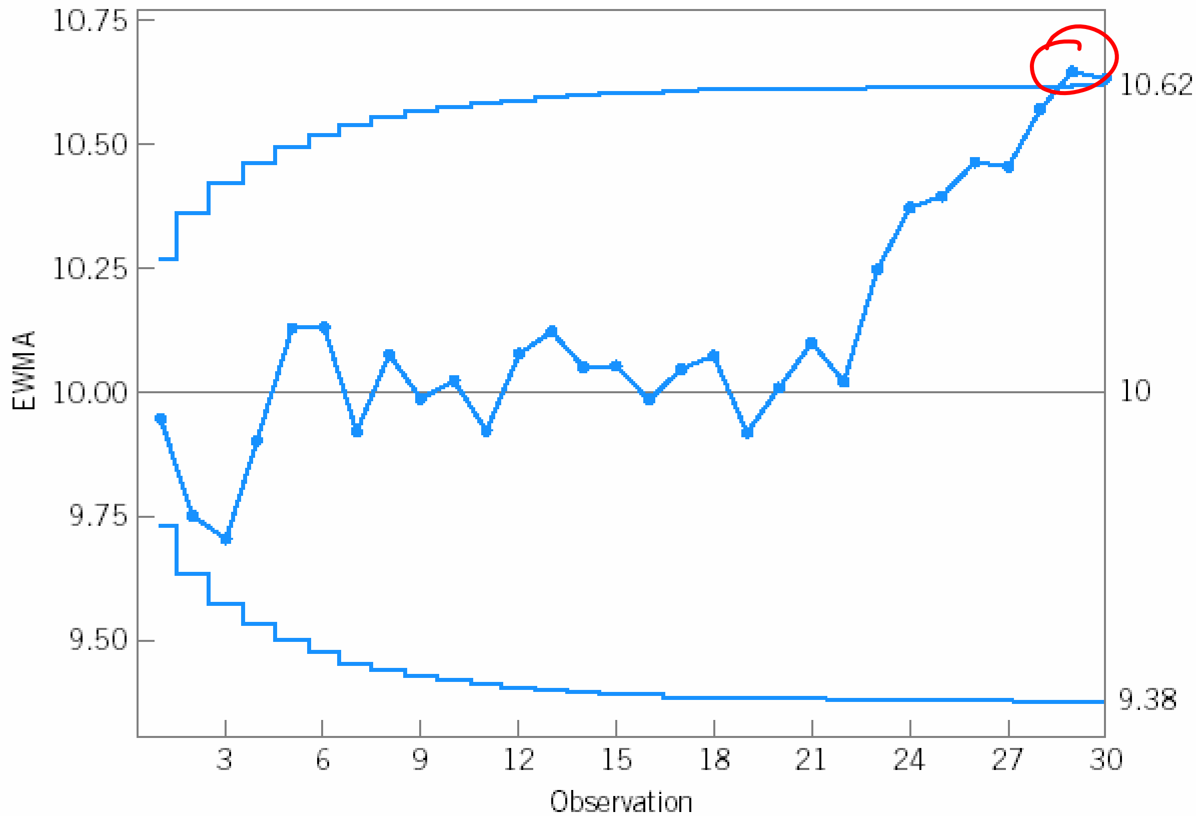


Figure 8-7 The EWMA control chart for Example 8-2.

EWMA Control Charts – Other Info

- EWMA control charts are insensitive to departures from normality of the quality measure.
- When rational subgroups of size n>1 are used replace

$$x_i \text{ with } \bar{x}_i \text{ and } \sigma \text{ with } \sigma_{\bar{x}} = \sigma / \sqrt{n}$$

$UCL = 1050 + 2.7 * 25\sqrt{\frac{.1}{1-.9}(1-.9^{2i})}$

$LCL = 1050 -$ "

In-Class Exercise Problem 8-17

- The target value of molecular weight is 1050 (μ_0) and the process standard deviation is thought to be about 25 (σ).
  - Set up an EWMA control chart for the mean of this process (find the control limits for i=1,2,3,4,5; find the steady state control limits, and compute the z_i for i=1,2,3,4,5). Use λ=0.1 and L=2.7.

$1-\lambda = 0.9$

$z_i = \lambda x_i + (1-\lambda) z_{i-1}$

i	x_i	z_i	UCL	LCL
1	1045	1049.5	1056.8	1048.3
2	1055	1050.1	1059.1	1040.9
3	1037	1048.8	1060.6	1039.4
4	1064	1050.3	1061.7	1038.3
5	1095	1054.7	1062.5	1037.5

$z_1 = .1 \times 1045 + .9 \times 1050$

Problem 8-17 Data

~~Obser.~~ SAMPLE #	Weight 1	2	3	4			
1	1045	1050	1065	1052			
2	1055						
3	1037						
4	1064						
5	1095						
6	1008						
7	1050						
8	1087						
9	1125						
10	1146						
11	1139						
12	1169						
13	1151						
14	1128						
15	1238						
16	1125						
17	1163						
18	1188						
19	1146						
20	1167						

THANK YOU