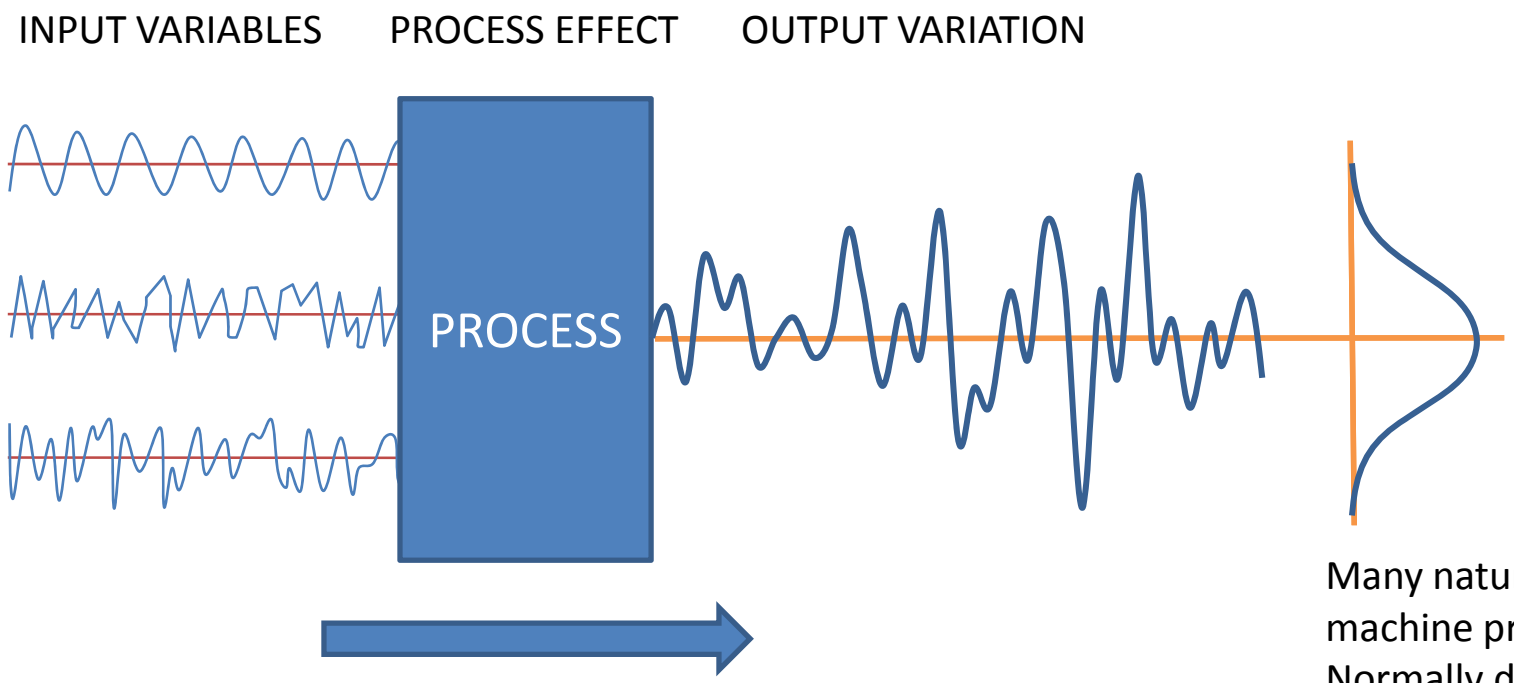
Statistical Process Control (SPC)

For Continuous and Discrete Data

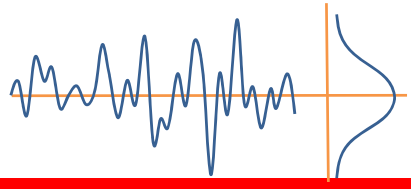
Process Variability

- Any process is subject to variation



Many natural & machine processes are Normally distributed (Bell shaped curve)

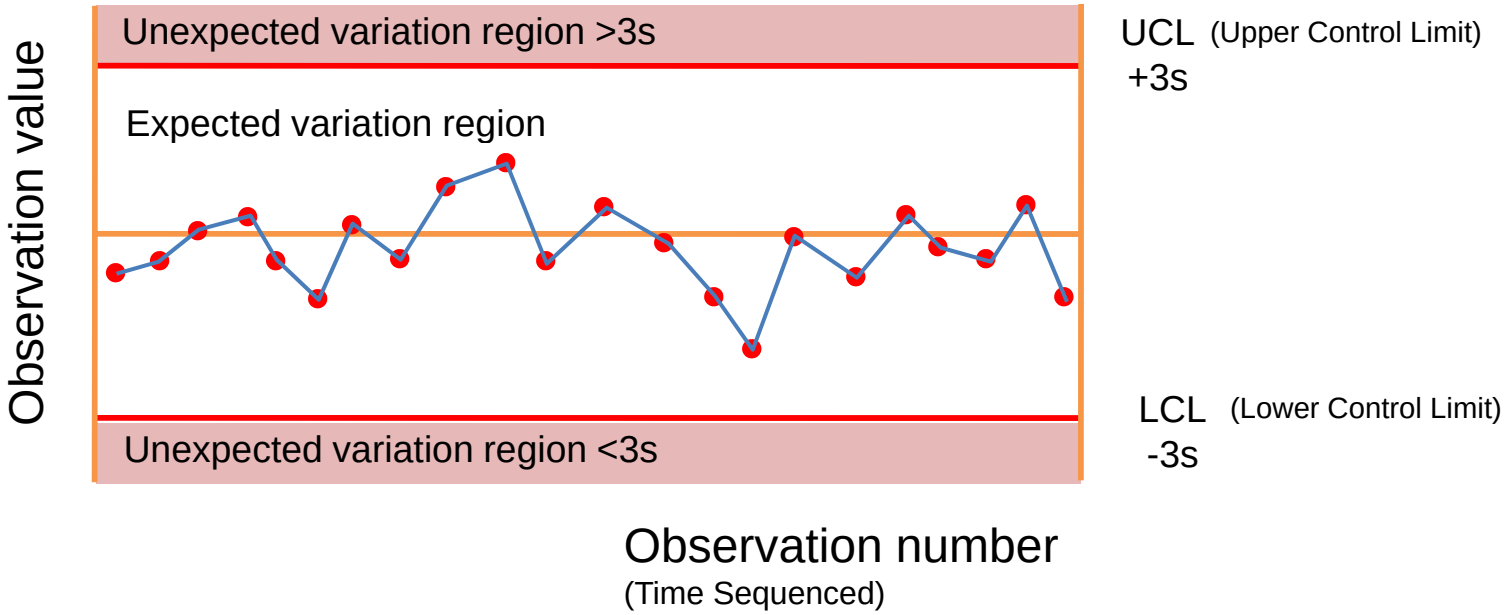
Process Stability



- “A process is said to be stable or controlled when, through the use of past experience, we can predict, at least between limits, how the process may be expected to behave in the future.”*
- Variations can be Common cause or Special cause
  - Common causes are from within the process, random and expected
  - Special causes are from outside the process, unstable and unexpected
- 99.73% of all data points measured from a process should be within ±3s (standard deviations) of the mean and have no trends or patterns present if Special causes are not present.

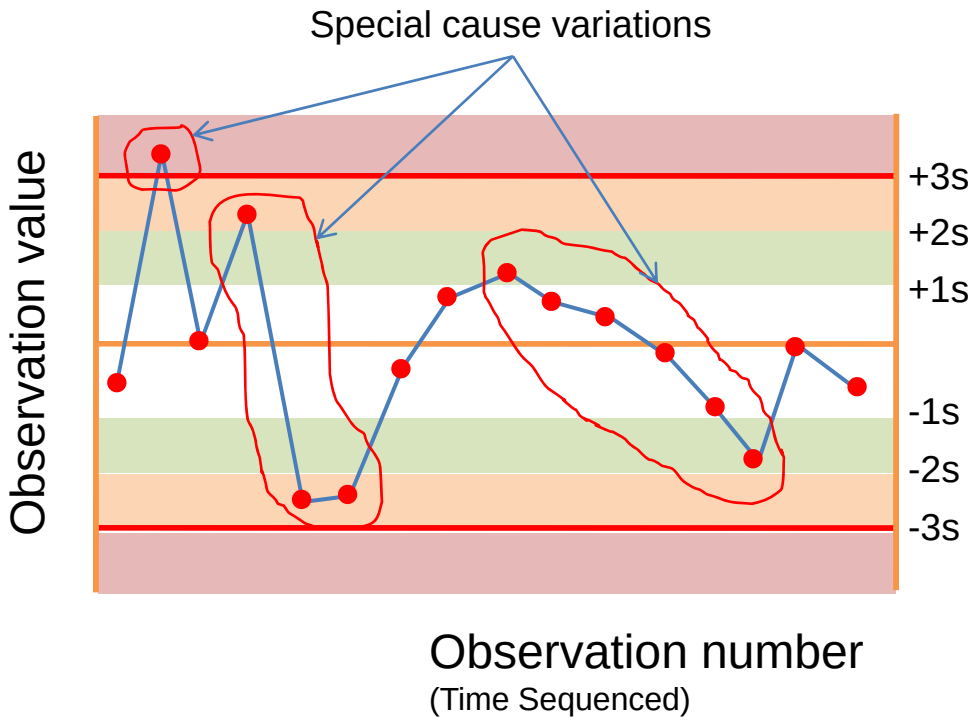
*Shewhart’s definition of a stable process. Considered to be the father of SPC.

Control Charting – a stable process



A stable process – no trends, patterns or unexpected large values

Control Charting – an unstable process



Tests for Special Causes

1. Any point outside control limits
2. 9 consecutive points on same side of centre line
3. 6 consecutive points increasing or decreasing
4. 2 of 3 points outside ±2s zones
5. 4 of 5 points outside ±1s zones
6. 14 consecutive points alternating up and down
7. 15 consecutive points within ±1s
8. 8 points in a row greater than ±1s, same side of centre line

An unstable process – with trends, patterns or unexpected large values

Charting Continuous Data

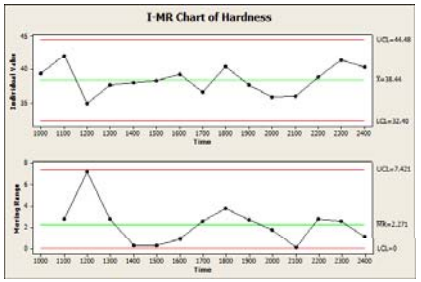
Continuous data has no boundaries between adjoining values e.g. time, temperature, weight

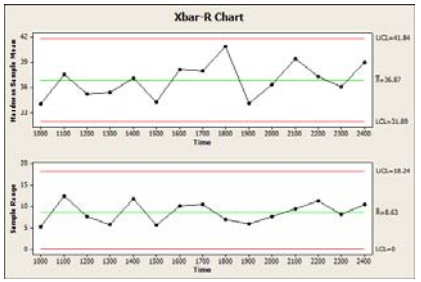
Chart Types

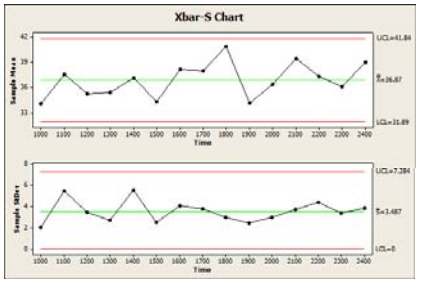
- I-MR Individual and Moving Range used when collecting more than 1 sample doesn't make sense e.g. destructive tests, low volume processes, batching

- Xbar-R Average and sample range for sample sizes less than 10

- Xbar-S Average and sample standard deviation for sample sizes10 or greater

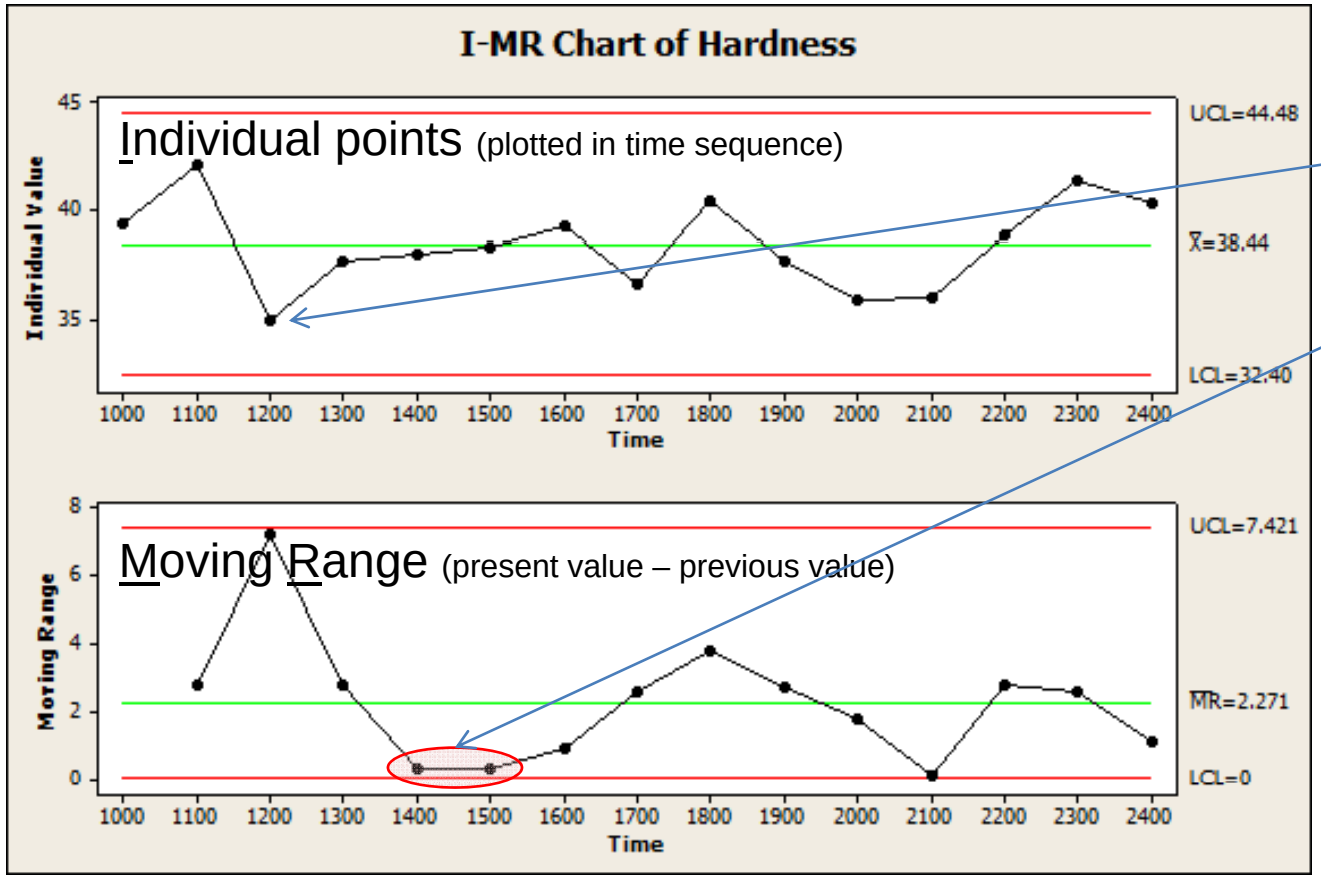






Constructing an I-MR chart

I-MR = Individual and Moving Range used when sub-groups are inappropriate e.g. destructive- tests.



Time	Hardness	MR
1000	39.3	
1100	42.1	2.8
1200	34.9	7.2
1300	37.7	2.8
1400	38	0.3
1500	38.3	0.3
1600	39.2	0.9
1700	36.6	2.6
1800	40.4	3.8
1900	37.7	2.7
2000	35.9	1.8
2100	36	0.1
2200	38.8	2.8
2300	41.4	2.6
2400	40.3	1.1
Average	**38.44**	**2.27**
StDev	**2.08**	**1.84**
UCL	**44.68**	**7.78**
LCL	**32.20**	**0.00**

N.B. UCL & LCL are calculated using Shewhart's variables and ±3s is approximate only

Notes:

UCL ≈ Average + 3s, LCL ≈ Average -3s

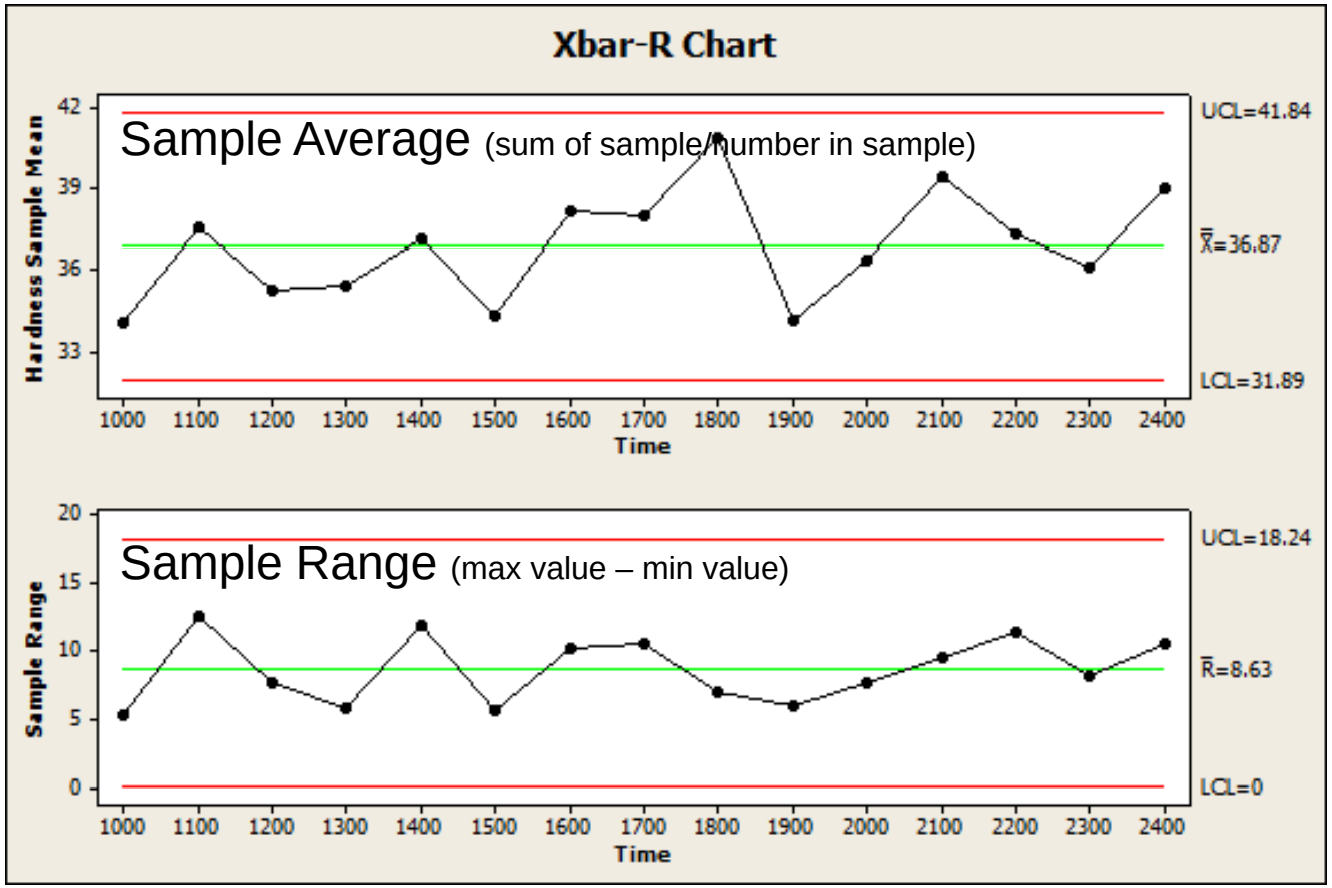
Use absolute values (no sign) when calculating moving range (in Excel =ABS(pres-prev))

LCL in moving range is set to zero if negative

Take a minimum of 25 points before constructing UCL and LCL lines

Constructing an Xbar-R chart

Xbar-R → Xbar(mean)=sample average, R= Range within sample, Sample sizes less than 9.



Time	Hardness	R
1000	34.05	5.32
1100	37.55	12.48
1200	35.26	7.55
1300	35.41	5.71
1400	37.15	11.80
1500	34.31	5.62
1600	38.14	10.18
1700	37.96	10.42
1800	40.87	7.01
1900	34.08	6.02
2000	36.34	7.65
2100	39.45	9.53
2200	37.31	11.39
2300	36.09	8.19
2400	39.05	10.46
Average	**36.87**	**8.62**
StDev	**2.05**	**2.43**
UCL	**43.02**	**15.91**
LCL	**30.72**	**0.00**

N.B. UCL & LCL are calculated using Shewhart's variables and ±3s is approximate only

Notes:

UCL ≈ Average + 3s, LCL ≈ Average -3s

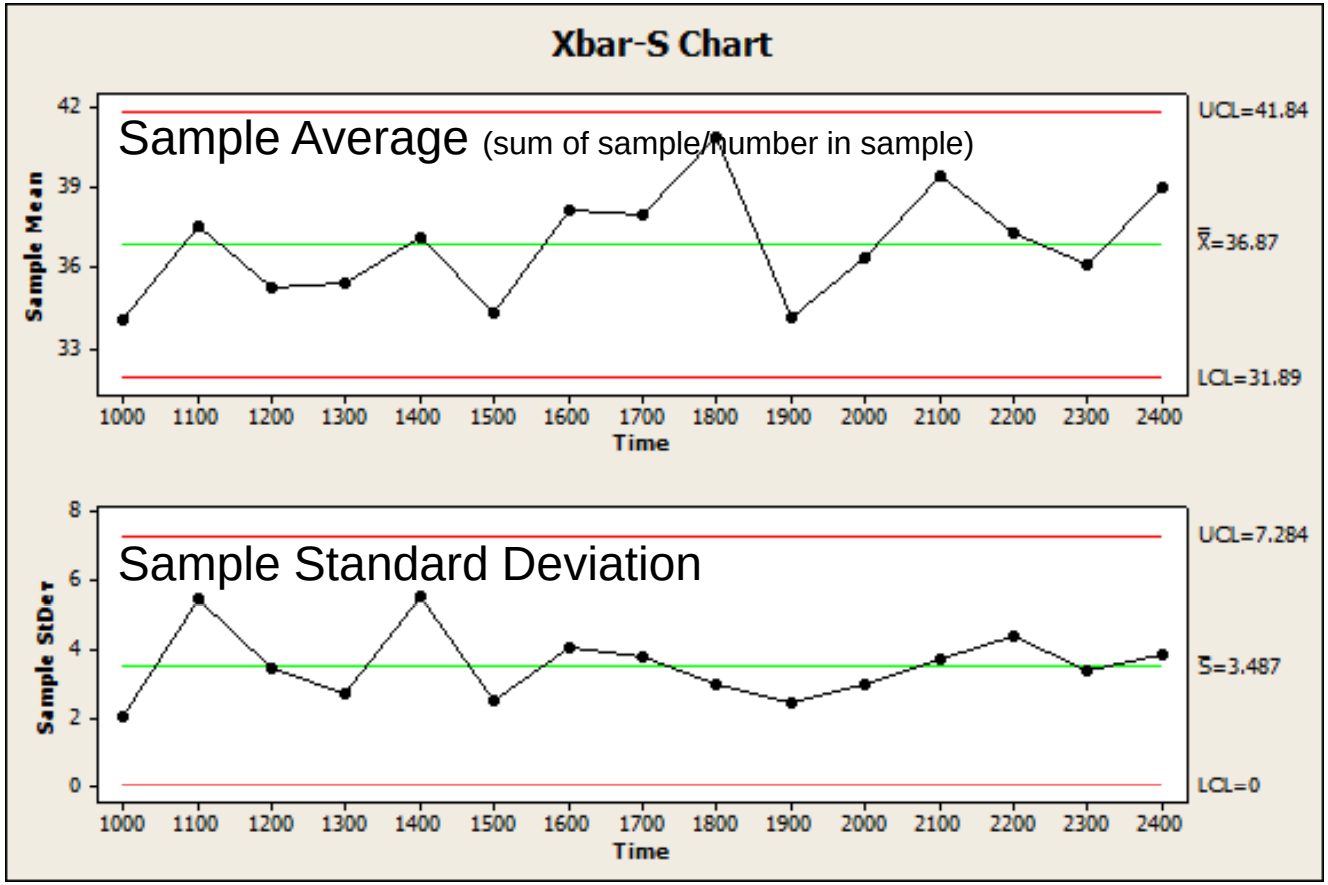
LCL in moving range is set to zero if negative

Xbar-R can be used on non-Normal data (the distribution of samples tend towards normality)

Take a minimum of 25 samples before constructing UCL and LCL lines

Constructing an Xbar-S chart

Xbar-S → Xbar(mean)=sample average, S= Standard Deviation within sample, Sample sizes 10 or greater.



Time	Hardness	S
1000	34.05	2.06
1100	37.55	5.49
1200	35.26	3.46
1300	35.41	2.74
1400	37.15	5.54
1500	34.31	2.50
1600	38.14	4.08
1700	37.96	3.76
1800	40.87	2.99
1900	34.08	2.43
2000	36.34	2.99
2100	39.45	3.73
2200	37.31	4.37
2300	36.09	3.41
2400	39.05	3.85
Average	**36.87**	**3.56**
StDev	**2.05**	**1.02**
UCL	**43.02**	**6.63**
LCL	**30.72**	**0.00**

N.B. UCL & LCL are calculated using Shewhart's variables and +-3s is approximate only

Notes:

UCL ≈ Average + 3s, LCL ≈ Average -3s

LCL in moving range is set to zero if negative

Xbar-S can be used on non-Normal data (the distribution of samples tend towards normality)

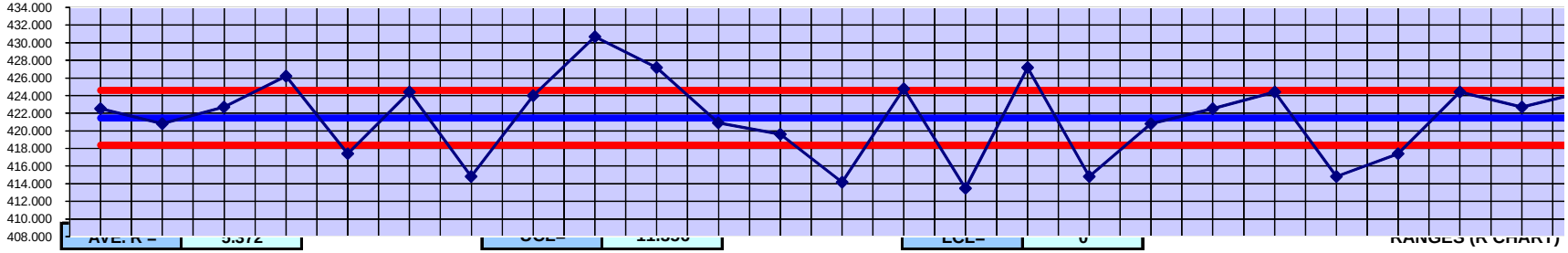
Xbar-R Chart Example

STATISTICAL PROCESS CONTROL CHART v1.04

Customer:	(from FORD Measurement System Software Calibration)			**Engineering Specification (Unit: mm)**						**PROCESS INFORMATION**		
Part Number:	LGPL4SCV10			**LSL:**	406.000	**SPEC.:**	424.000	**USL:**	442.000	**Significant trends of data points:**	**X BAR Chart**	**R Chart**
Item Key:	SAMPLE			**MINUS:**	18.000	**NOMINAL:**	424.000	**PLUS:**	18.000	Increasing — RUN LENGTH	4	5
Characteristic:	SAMPLE			**GAUGE USED**		**SAMPLE SIZE/FREQ.**		**Readings/Subgroup**		Increasing — HOW MANY RUNS	1	1
Process:	SAMPLE							5		Decreasing — RUN LENGTH	5	3
Date:	SAMPLE									Decreasing — HOW MANY RUNS	1	2
PREPARED:		**CHECKED:**		**APPROVED:**		**CONTROL**	Xbar & R			**Out of Control Limits**	12	3
DATE:		**DATE:**		**DATE:**		**CHART:**				**Consecutive data points above avg.**	3	5
◉ Two sided spec (bilateral)		○ One sided (MIN)		○ One sided (MAX)						**Consecutive data points below avg.**	3	9

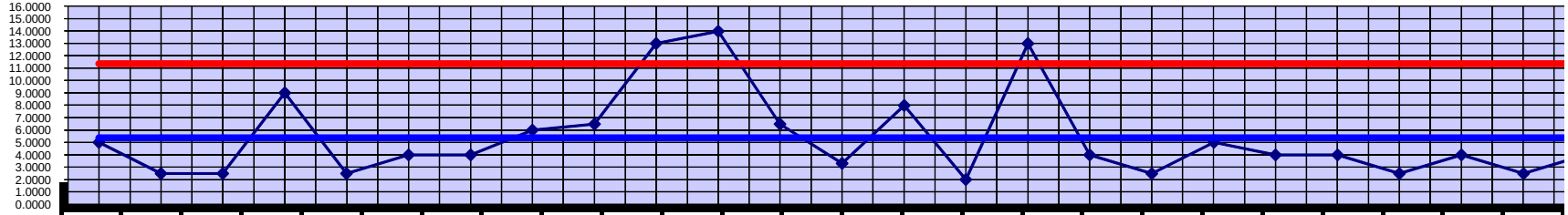
AVE. Xbar =	421.479	**UCL=**	424.579	**LCL=**	418.380

AVERAGE (X BAR CHART)



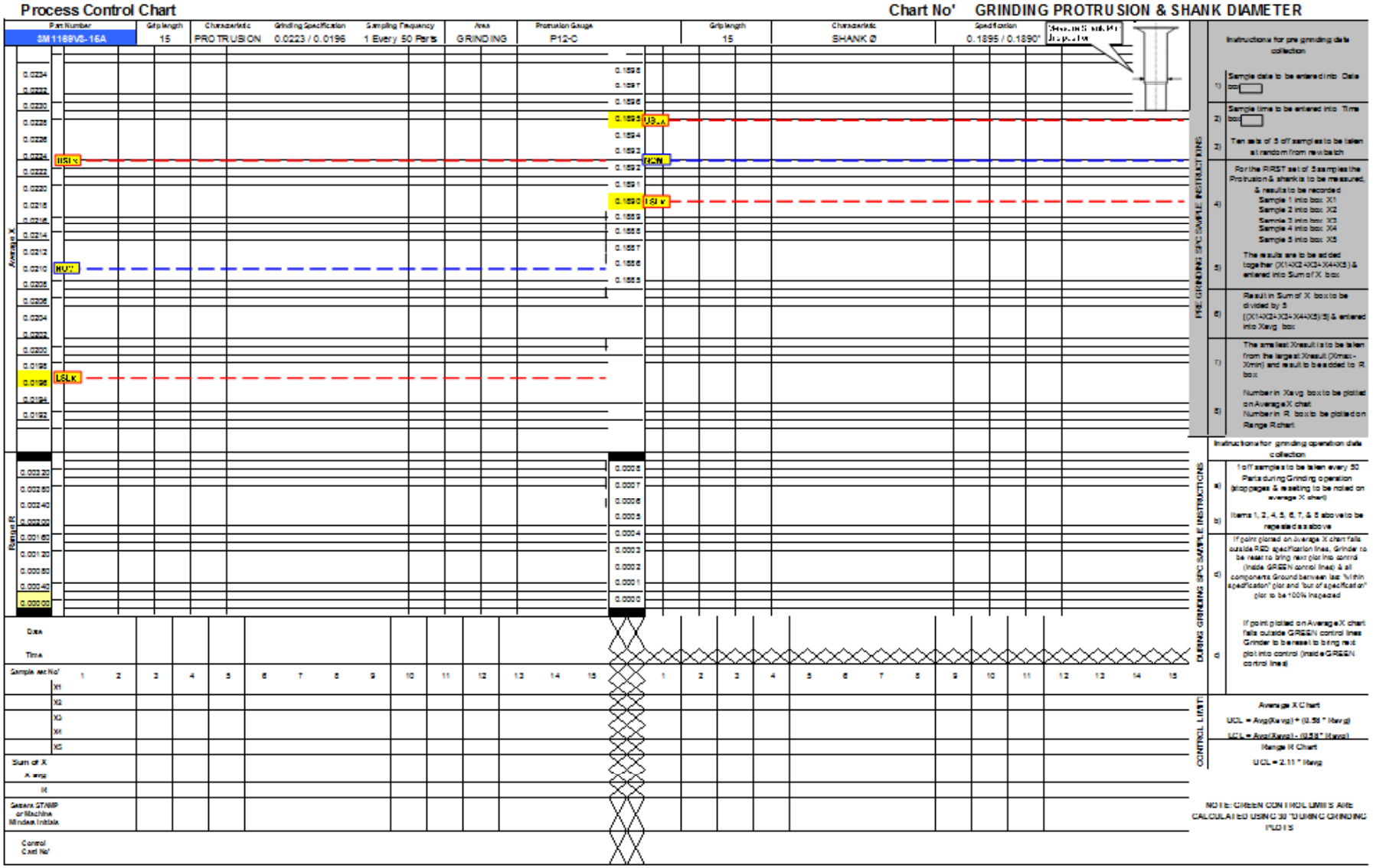
AVE. R =	5.372	**UCL=**	11.696	**LCL=**	0

RANGES (R CHART)

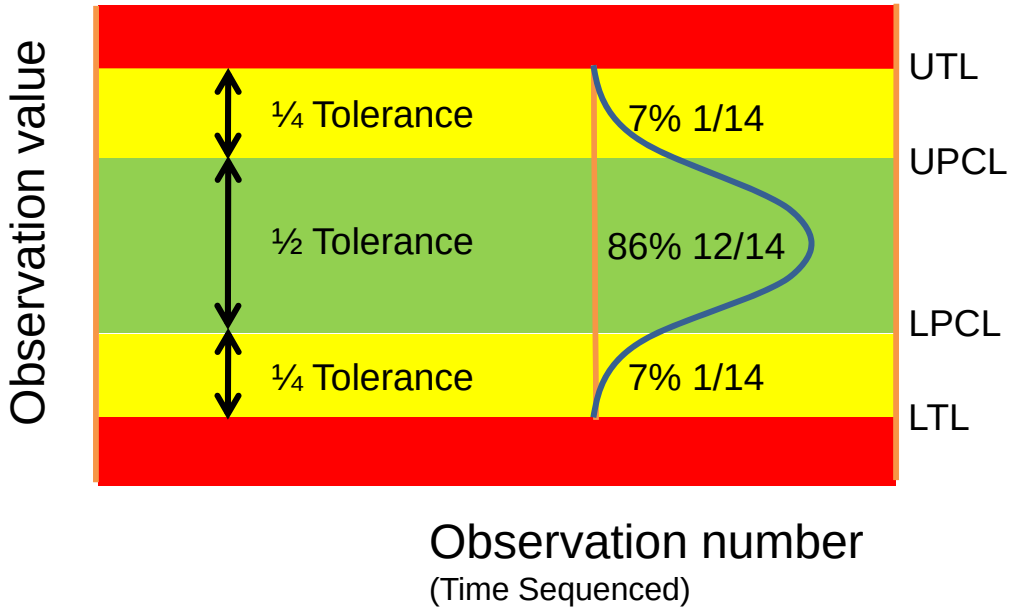


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
DATE/TIME																									
READINGS 1	425.0000	422.0000	423.0000	427.0000	418.5000	425.0000	415.5000	422.5000	429.0000	429.8000	416.0000	420.5000	415.3000	428.5000	413.0000	429.8000	415.5000	422.0000	425.0000	425.0000	415.5000	418.5000	425.0000	423.0000	425.0000
2	421.5000	421.5000	421.5000	427.0000	417.0000	426.0000	414.5000	425.0000	428.5000	428.5000	420.5000	423.0000	414.5000	421.5000	415.0000	428.5000	414.5000	421.5000	421.5000	426.0000	414.5000	417.0000	426.0000	421.5000	426.0000
3	420.0000	420.5000	422.0000	420.0000	418.0000	424.0000	417.0000	426.5000	434.5000	421.5000	420.5000	419.0000	415.0000	428.0000	413.5000	421.5000	417.0000	420.5000	420.0000	424.0000	417.0000	418.0000	424.0000	422.0000	424.0000
4	422.0000	419.5000	423.0000	429.0000	416.0000	425.0000	414.0000	420.5000	428.0000	434.5000	430.0000	416.5000	414.0000	420.5000	413.0000	434.5000	414.0000	419.5000	422.0000	425.0000	414.0000	416.0000	425.0000	423.0000	425.0000
5	424.0000	420.5000	424.0000	428.0000	417.5000	422.0000	413.0000	425.5000	433.5000	421.5000	417.5000	419.0000	412.0000	425.5000	413.0000	421.5000	413.0000	420.5000	424.0000	422.0000	413.0000	417.5000	422.0000	424.0000	422.0000
AVE .X =	422.5000	420.8000	422.7000	426.2000	417.4000	424.4000	414.8000	424.0000	430.7000	427.1600	420.9000	419.6000	414.1600	424.8000	413.5000	427.1600	414.8000	420.8000	422.5000	424.4000	414.8000	417.4000	424.4000	422.7000	424.4000
R =	5.0000	2.5000	2.5000	9.0000	2.5000	4.0000	4.0000	6.0000	6.5000	13.0000	14.0000	6.5000	3.3000	8.0000	2.0000	13.0000	4.0000	2.5000	5.0000	4.0000	4.0000	2.5000	4.0000	2.5000	4.0000

Xbar-R Chart Example



SPC PRE-control – a simple control method



Average six sample pairs between consecutive adjustments

Key:

UTL	– Upper Tolerance Limit
LTL	– Lower Tolerance Limit
UPCL	– Upper Pre-Control Limit
LPCL	– Lower Pre-Control Limit

To qualify set-up:

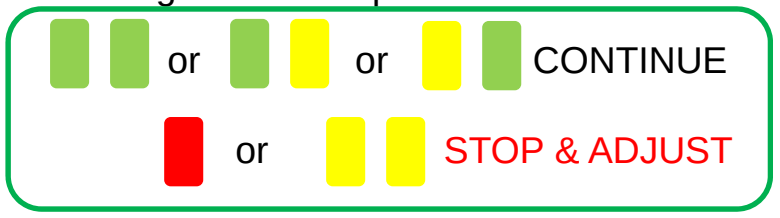
- If five consecutive pieces are in Green zone, set-up is ok to run
- If one yellow, restart counting
- If two consecutive yellows, adjust the process
- If one reading is red, adjust the process

Qualifying setting

Sample two consecutive pieces A and B

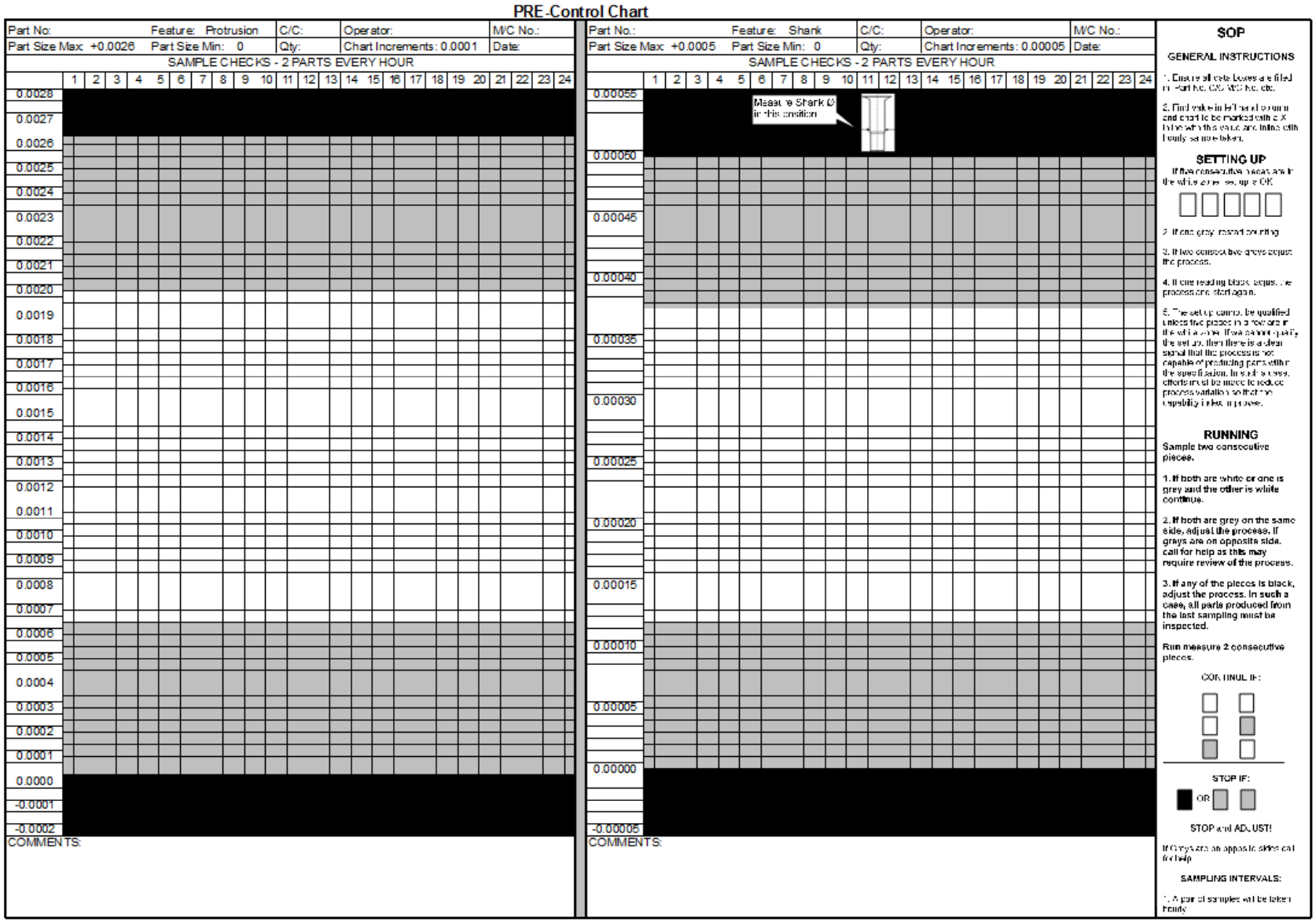
- If both are green or one is yellow and the other is green, continue
- If both A and B are yellow on the same side, adjust the process. If yellows are on the opposite sides, call for help as this may require review of the process
- If any of the pieces is red, adjust the process. In such a case, parts produced from the last sampling must be inspected

Running measure 2 pieces in a row



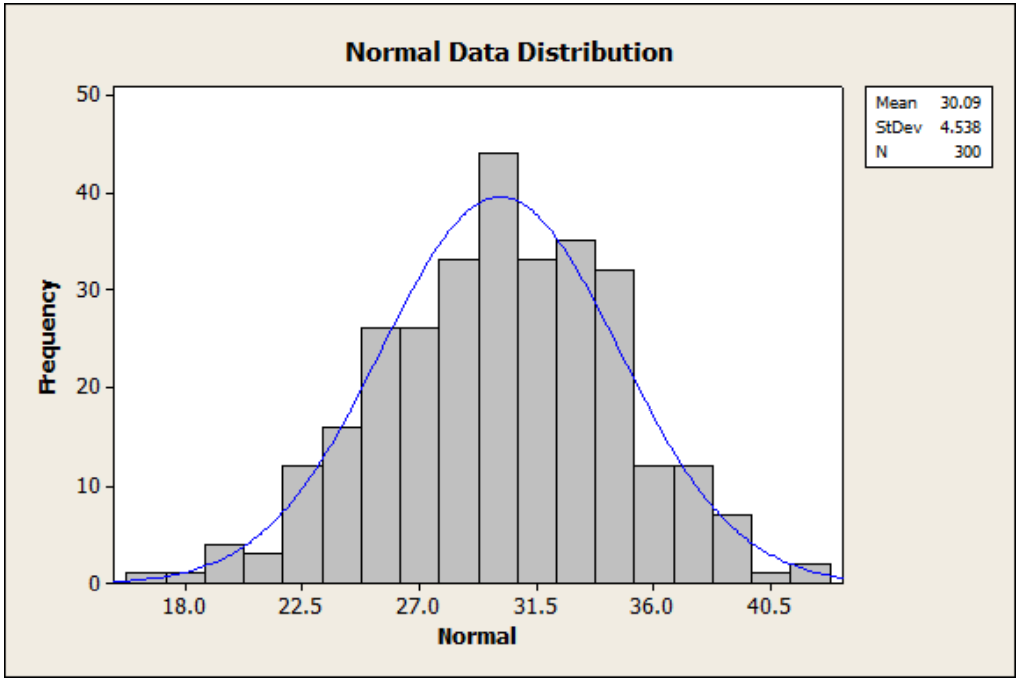
* Paper by Hemant P. Urdhwareshe www.symphonytech.com

Example of PRE-Control chart

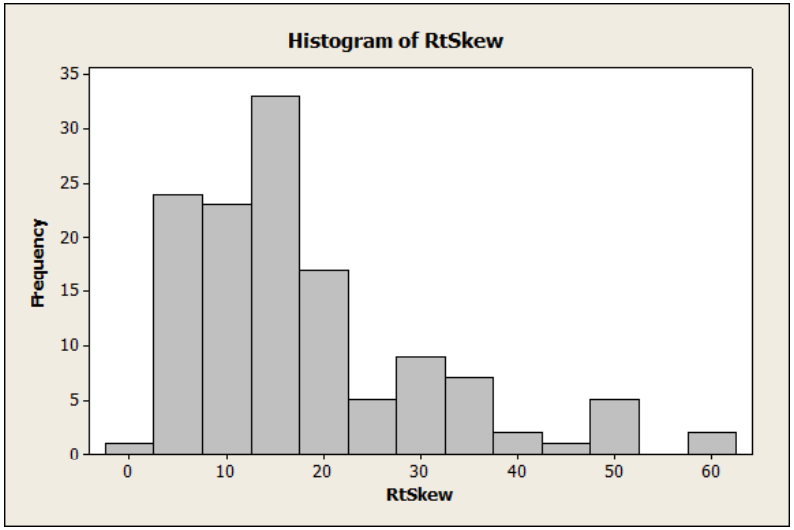


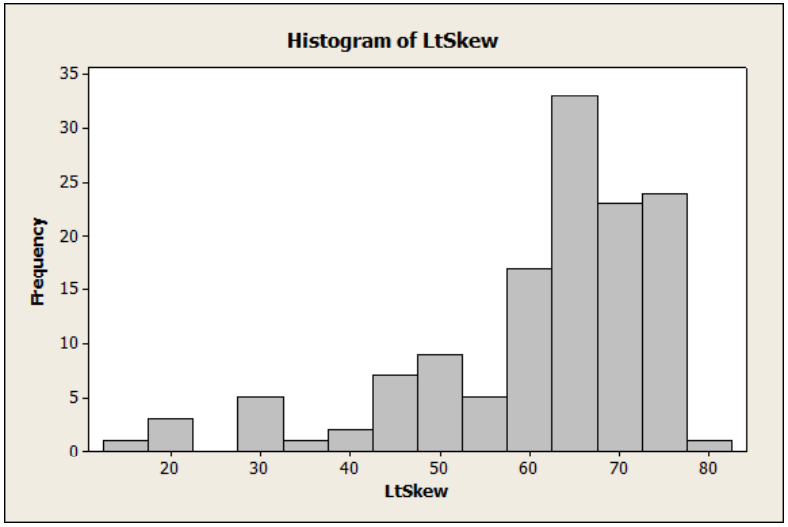
Normal vs Non-normal Data

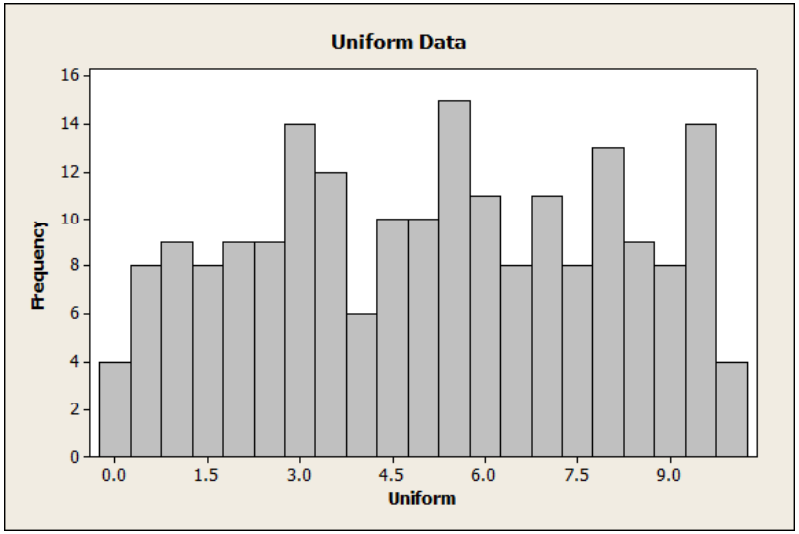
- Data should be Normally distributed when using I-MR charts
- Data should be preferably Normal for Xbar-R charts however the means of samples from any population will tend towards Normal according to the Central Limit Theorem.

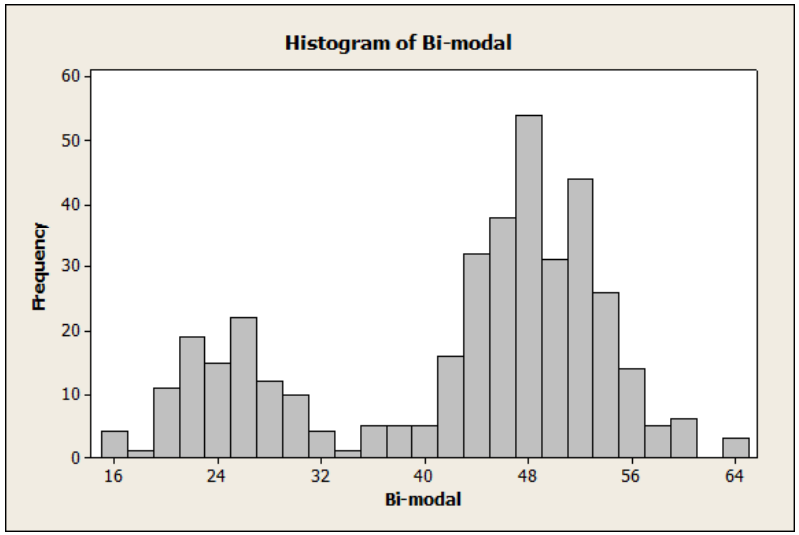


Non-Normal Data









SPC for Discrete (Attribute) Data

Counts, ranks, names, defects, defectives

Charting Discrete (Attribute) Data

Discrete data has clear boundaries between adjoining values includes names, categories, counts and rank orders e.g. dates, colours, defects, defectives

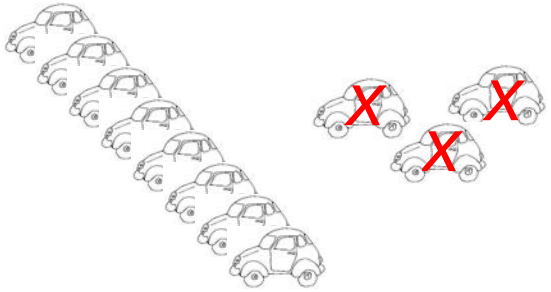


Chart Types

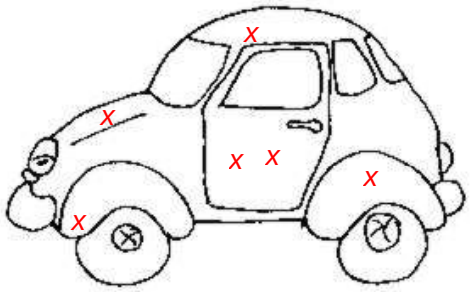
- NP Charts – plots the ***number*** of defectives per lot or subgroup size (constant lot sizes)
- P Charts – plots the ***proportion*** of defectives per lot or subgroup (variable lot sizes)

Defectives

- C Charts – plots the number of defects from a lot or subgroup (constant lot sizes)
- U Charts – plots the number of defects per unit sampled (variable lot sizes)

Defects

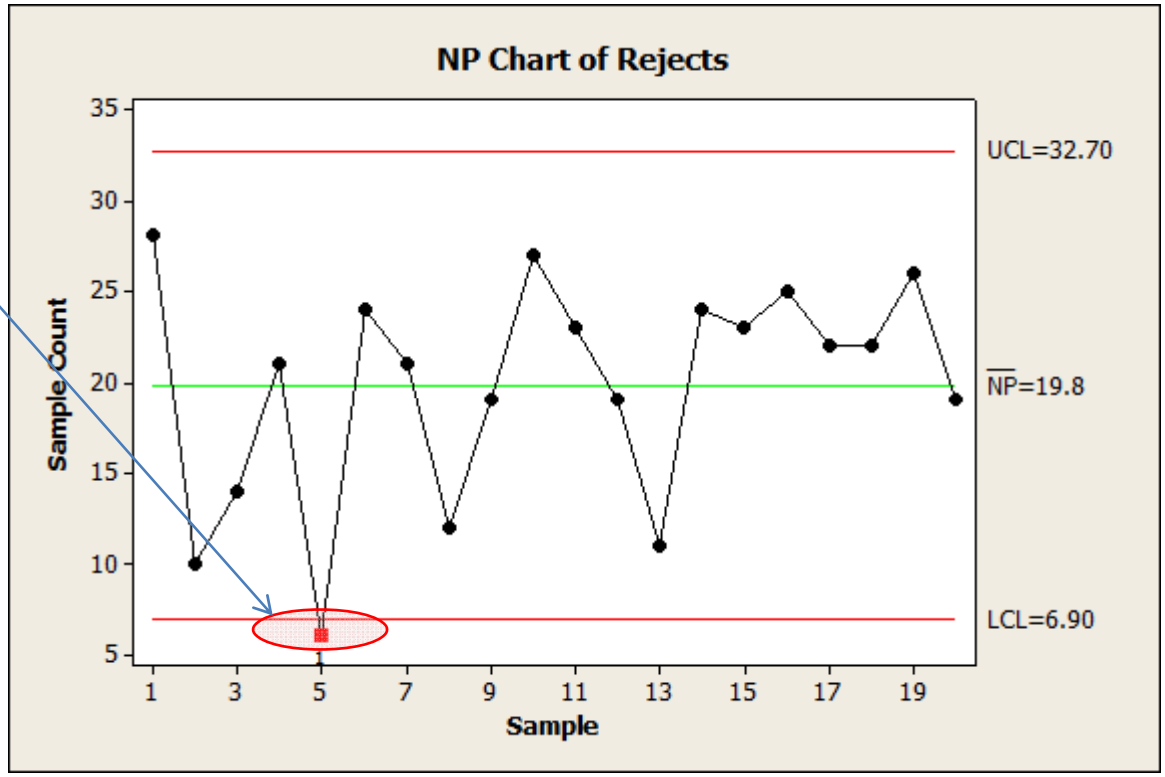
	Defectives	Defects
Variable Sample	P-Chart	U-Chart
Constant Sample	NP-Chart	C-Chart



NP Charts – Defectives per even lot size

Constant sub-group size

Days	Units	Rejects
1	300	28
2	300	10
3	300	14
4	300	21
5	300	6
6	300	24
7	300	21
8	300	12
9	300	19
10	300	27
11	300	23
12	300	19
13	300	11
14	300	24
15	300	23
16	300	25
17	300	22
18	300	22
19	300	26
20	300	19
Average		**19.80**
StDev		**6.13**
UCL		**38.18**
LCL		**1.42**



Uses: number of defective parts in a constant sample size or number of unacceptable calls from a sample of size of 50 (from hundreds of calls per day) phone calls monitored per day

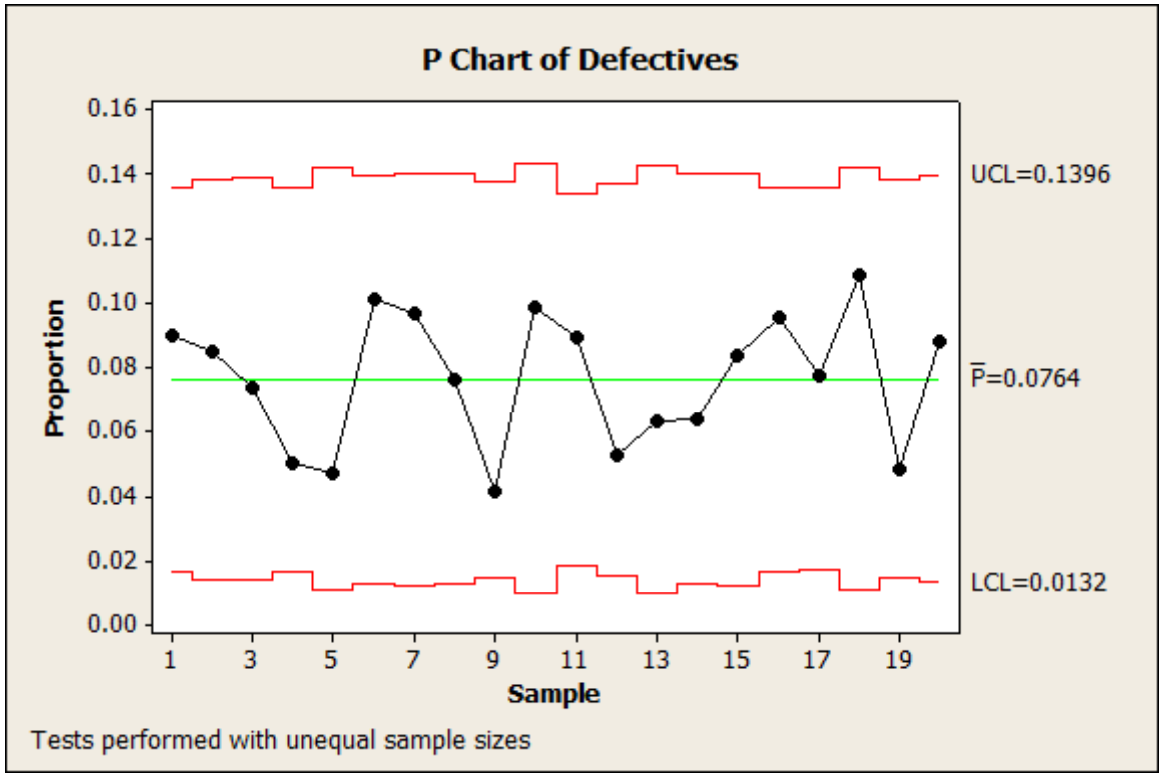
Use specialist SPC software to calculate UCL & LCL values such as SPC for Excel or Minitab

Use ±3s from average as an approximation

P Chart – Proportions variable lot sizes

Day	Order	Defectives	Proportion
1	178	16	0.090
2	165	14	0.085
3	162	12	0.074
4	179	9	0.050
5	148	7	0.047
6	158	16	0.101
7	155	15	0.097
8	157	12	0.076
9	168	7	0.042
10	142	14	0.099
11	190	17	0.089
12	171	9	0.053
13	143	9	0.063
14	156	10	0.064
15	155	13	0.084
16	178	17	0.096
17	181	14	0.077
18	147	16	0.109
19	166	8	0.048
20	159	14	0.088
Average		**12.45**	**0.077**
StDev		**3.38**	**0.021**
UCL		**22.59**	**0.138**
LCL		**2.31**	**0.015**

Varying sub-group size. Subgroup sizes >50



Uses: 30% of orders received per day (orders vary from 300 to 600) are evaluated as shipped or not shipped.

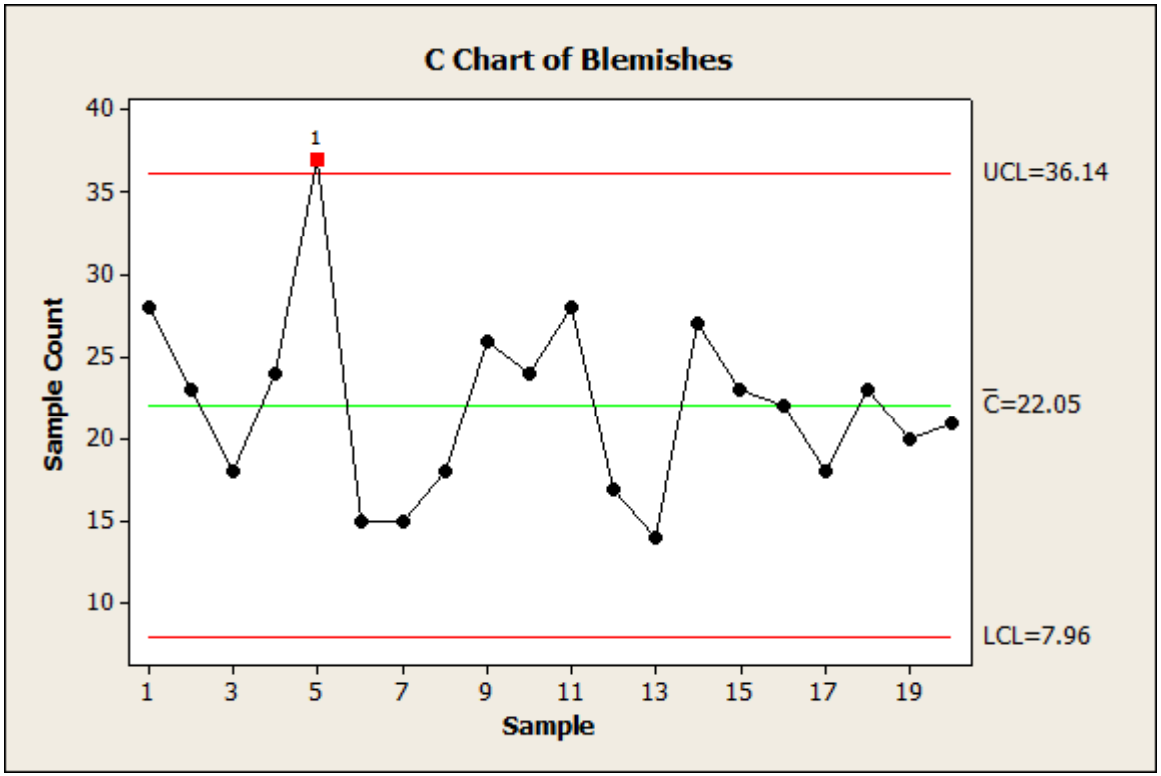
Use specialist SPC software to calculate UCL & LCL values such as SPC for Excel or Minitab

Use ±3s from average as an approximation

C Charts – defects in a constant lot

Constant sub-group size i.e. 1

Batch	Blemishes
1	28
2	23
3	18
4	24
5	37
6	15
7	15
8	18
9	26
10	24
11	28
12	17
13	14
14	27
15	23
16	22
17	18
18	23
19	20
20	21
Average	**22.05**
StDev	**5.57**
UCL	**38.75**
LCL	**5.35**



Uses: errors in a form with a constant number of fields or blemishes on one type of car

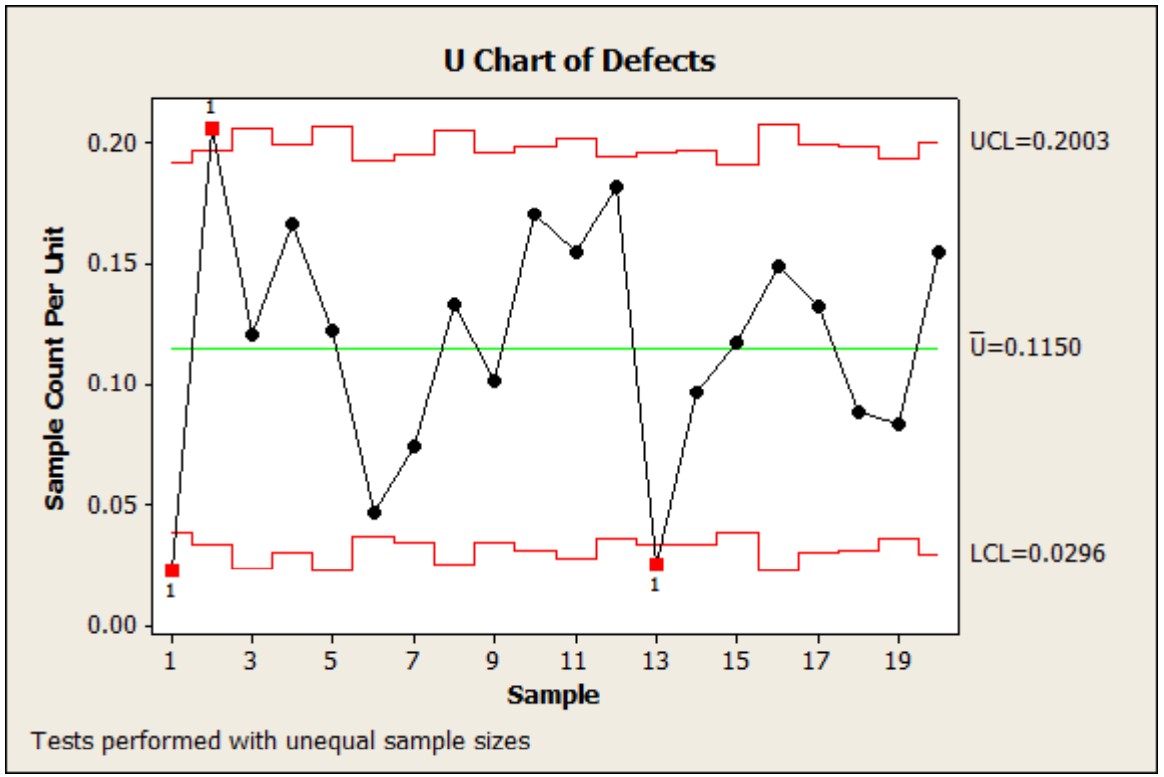
Use specialist SPC software to calculate UCL & LCL values such as SPC for Excel or Minitab

Use ±3s from average as an approximation

U Chart – defects per unit sampled

Varying sub-group size

Day	Output	Defects	Proportion
1	176	4	0.023
2	155	32	0.206
3	124	15	0.121
4	144	24	0.167
5	123	15	0.122
6	171	8	0.047
7	161	12	0.075
8	128	17	0.133
9	158	16	0.101
10	147	25	0.170
11	136	21	0.154
12	165	30	0.182
13	157	4	0.025
14	155	15	0.097
15	179	21	0.117
16	121	18	0.149
17	144	19	0.132
18	147	13	0.088
19	168	14	0.083
20	142	22	0.155
Average		**17.25**	**0.117**
StDev		**7.43**	**0.050**
UCL		**39.55**	**0.269**
LCL		**0.00**	**0.000**



Uses: number of errors on forms with variable number of fields or blemishes on vehicles of different surface areas

UCL & LCL is set to zero if negative

Use specialist SPC software to calculate UCL & LCL values such as SPC for Excel or Minitab

Use ±3s from average as an approximation

Choosing the Correct Control Chart

Continuous Data

- I-MR Individual and Moving Range used when collecting more than 1 sample doesn't make sense e.g. destructive tests, low volume processes, batching
- Xbar-R Average and sample range for sample sizes less than 10
- Xbar-S Average and sample standard deviation for sample sizes10 or greater

Discrete or Attribute Data

- NP Charts – plots the ***number*** of defectives per lot or subgroup size (constant lot sizes)
- P Charts – plots the ***proportion*** of defectives per lot or subgroup (variable lot sizes)
- C Charts – plots the number of defects from a lot or subgroup (constant lot sizes)
- U Charts – plots the number of defects per unit sampled (variable lot sizes)

Points to Note

- SPC can be expensive to implement. Work out it’s value to you and how to minimise the workload
- Operators and support staff must be properly trained or SPC won’t work
- Action must be taken for out of control situations or SPC is useless
- A process can be in control but still have poor capability and rejects.