

# Statistical

Process Control Cerming



Quality & Productivity Society of Pakistan



#### **Contents**

- Quality & TQM
- Basic Statistics
- Seven QC Tools
- Control Charts
- Process Capability Analysis



# **CUSTOMERS**





# "Anyone who thinks customers are not important should try doing without them for a week"

Source: Unknown





- External Customers
  - Final Customers/End-Users
- Internal Customers



## **Types of Customers**

"The next operation as customer"

- Kaoru Ishikawa



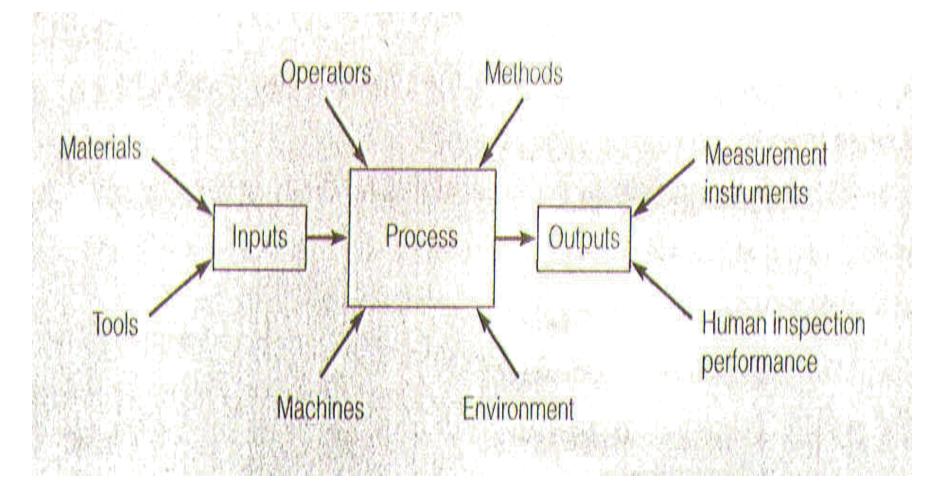
#### **Types of Customers**

#### **Exercise**

- External Customers 3 main customers (describe type)
- Internal Customers 3 main customers

# Sources of Variation in Production Processes







# What is Quality?



# Quality





#### **Fitness for Use**



(Juran 1988)



# **Quality in goods**

- Performance
- Features
- Durability
- Reliability
- Conformance
- Serviceability
- Aesthetics
- Perceived Quality



## **Quality in Services**

- Tangibles
- Reliability
- Responsiveness
- Competence
- Courtesy
- Security
- Access; Communication &Understanding the Customer

# What Is Quality? The Experts Say...

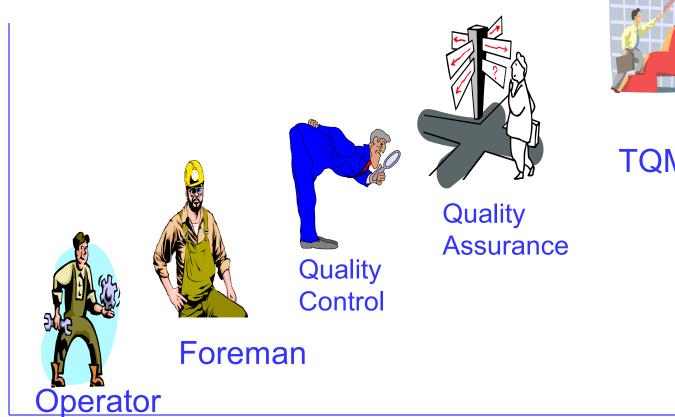


- Conformance to requirements (Philip B. Crosby)
- Zero Defects (Philip B. Crosby)
- Fitness for use (Joseph M. Juran)
- Reduced variation (W. Edwards Deming)

# **Quality Control Evolution**









**TQM** 



# **Total Quality** Management, earning for





# **Total Quality Management**

Achieve customer satisfaction through continually improving all work process and participation of employees.





- Leadership
- Employee Involvement
- Product/Process Excellence
- Customer Focus

# **Major Contributors to the** development of TQM

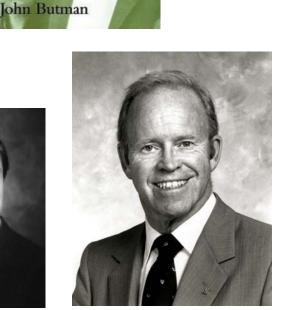
- Dr Edwards Deming
- Dr Joseph Juran
- Philip Crosby
- Armand Feigenbaum
- Prof. Kaori Ishikawa
- Genichi Taguchi
- Musaaki Imai





Lifetime

Influence



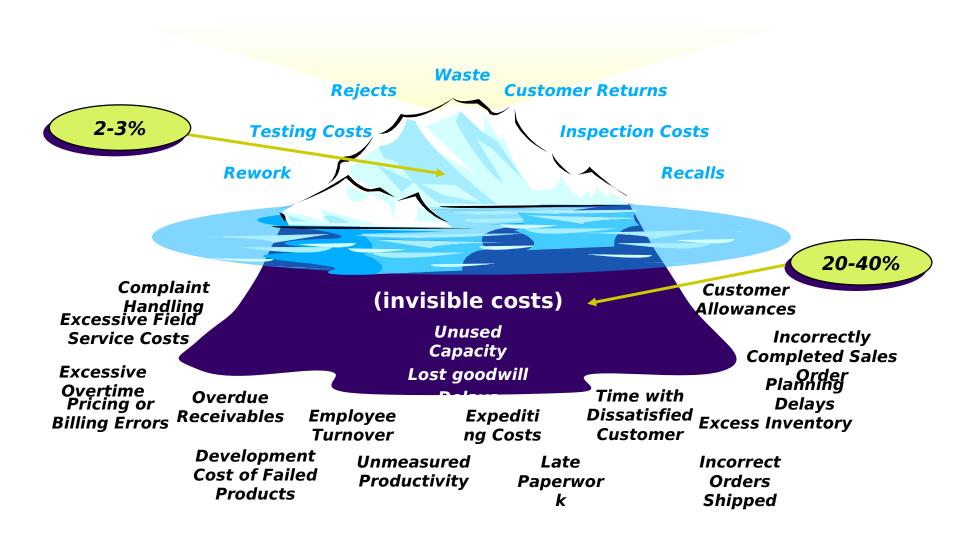






#### Variation results in cost







# **Basic Statistics**



# **Population**



Any well-defined group of individuals whose characteristics are to be studied.

Students of a college

**Books in Library** 

Shirts in Market

Fishes in Lake

# Sample



Part of the population which is to be studied.

#### **Variable**



Characteristics of the individuals of a population or sample which varies from individual to individual.

Marks obtained by Student

Height of Students

Temperature of Person

**Dimensions of Product** 

#### **Statistics**



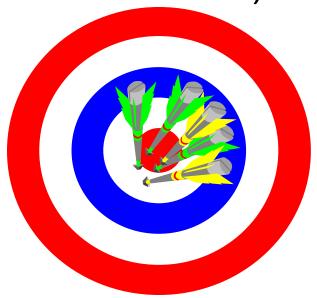
Statistics are numericals in any field of study.

Statistics deals with techniques or methods for collecting, analysing and drawing conclusions from data.

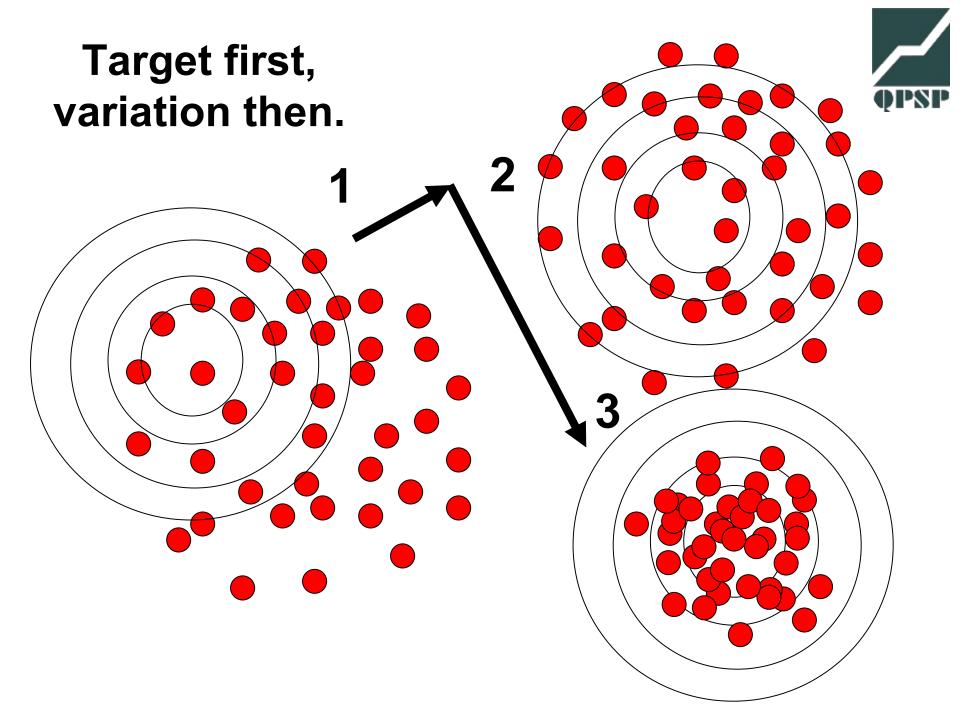
#### **ACCURATE & PRECISE**

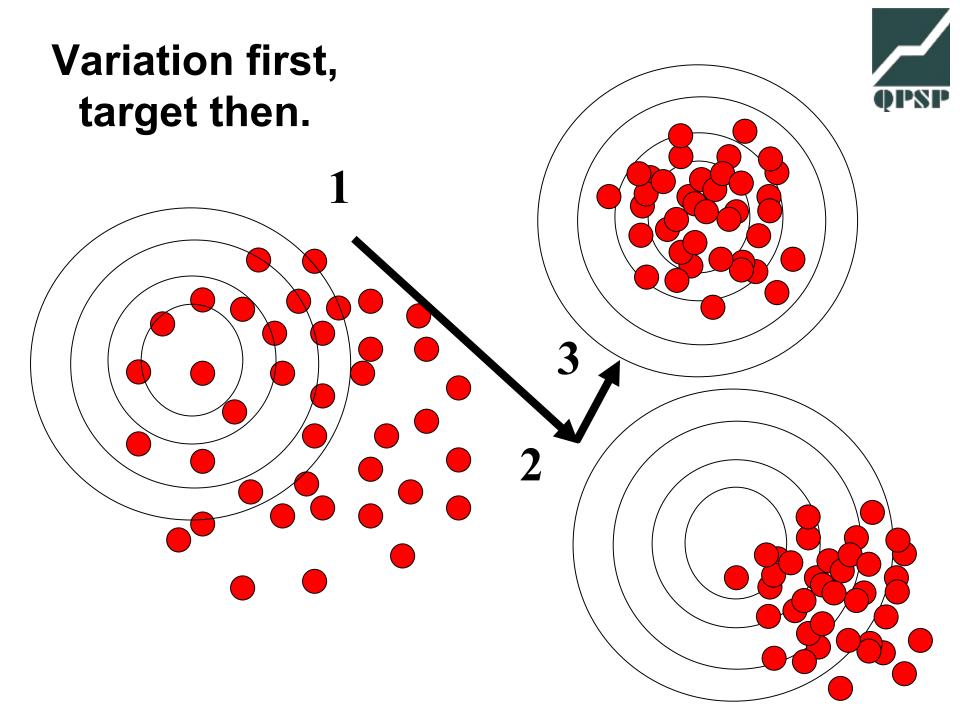


VERY CLOSE TOGETHER (LOW VARIATION) AND CENTERED ON TARGET (TRUE VALUE)



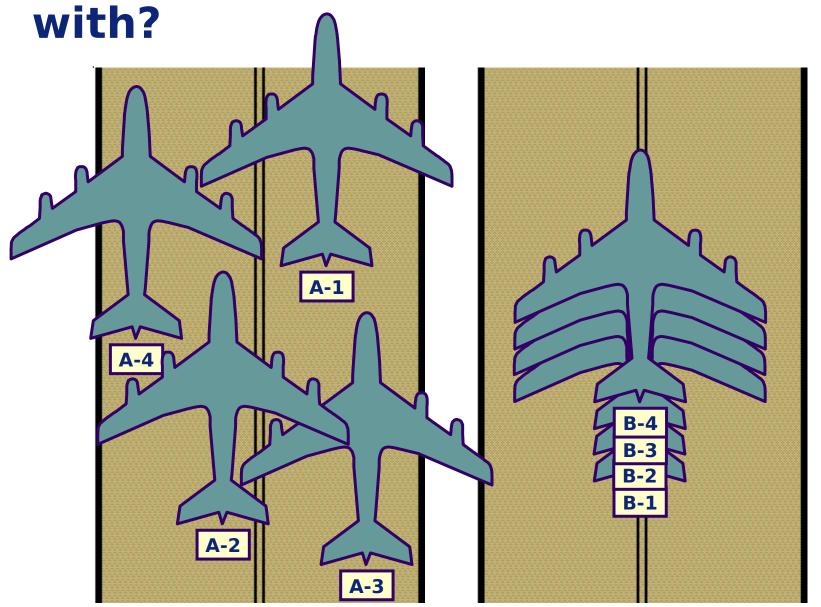
THE GOAL OF ANY PROCESS
PRECISE AND ACCURATE





Which pilot do you want to fly







#### **Quality Engineering Terminology**

#### **Specifications**

Quality characteristics being measured are often compared to standards or specifications.

- Nominal or target value
- Upper Specification Limit (USL)
- Lower Specification Limit (LSL)



# Quality Engineering Terminology

- When a component or product does not meet specifications, they are considered to be *nonconforming*.
- A nonconforming product is considered defective if it has one or more defects.
- Defects are nonconformities that may seriously affect the safe or effective use of the product.



#### Types of Data

#### **Variables**



#### "Things we measure"

- Length
- Height
- Weight
- Volume

Time

- Temperature
- Diameter
- Tensile Strength
- Strength of Solution

#### **Attributes**



#### "Things we count"

- Number or percent of defective items in a lot.
- Number of defects per item.
- Types of defects.
- Value assigned to defects (minor=1, major=5, critical=10)

#### **Averages**



Mean, median and mode

Weekly rent paid by 15 students sharing accommodation, 1998 (£)				
45	35	51	45	49
51	40	42	46	36
37	42	47	49	42

Mean (or average)

add observations and divide by number of observations 657/15 = 43.8

$$\frac{1}{x} = \frac{\sum x}{n}$$



#### Averages... 2

- Median the middle observation.
  - Arrange the observations and find the middle one (n+1)/2th observation
    - 35 36 37 40 42 42 42 45 45 46 47 49 49 51 51

the 8th observation (15+1)/2 is <u>45</u>

Mode – the most frequent observation

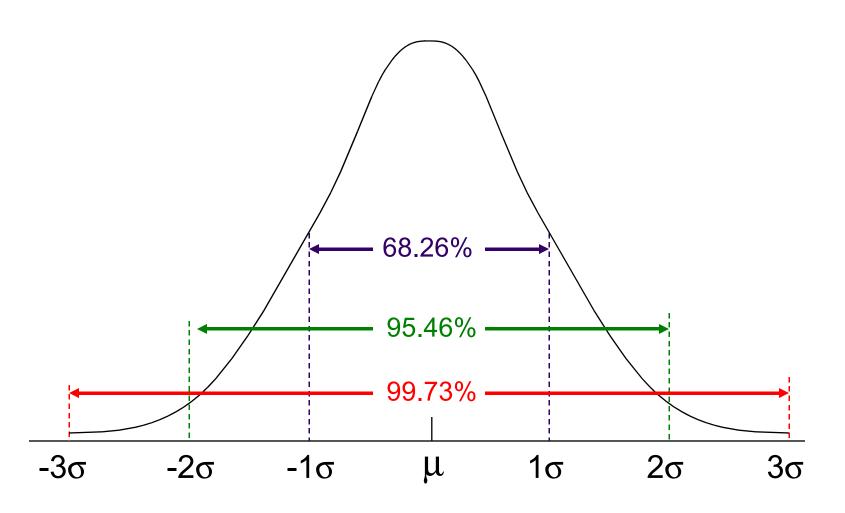
#### MEASURES OF DISPERSION



The dispersion is defined as the scatter or spread of the values from one another or from some common value.



#### MEASURES OF DISPERSION



### MEASURES OF DISPERSION



#### ALTERNATIVE TO CENTRAL TENDENCY

- RANGE (R): HIGHEST LOWEST [Max Min]
- VARIANCE: How data is spread out, about the mean.

$$s^2 = \frac{\sum (x - \overline{\chi})^2}{n - 1}$$

• STANDARD DEVIATION: Positive Square Root of Variance.

$$S = \sqrt{\frac{\sum (x - \overline{x})^2}{n - 1}}$$

## **Spreads**



- Standard Deviation (SD) calculated as below
  - calculate residuals individual observation minus mean square and sum these
  - divide by number of observations minus 1 [gives Variance]
  - take square root for Standard Deviation

$$SD = \sqrt{\frac{\sum (Y_i - \overline{Y})^2}{n-1}}$$

- example peoples heights (cm)
  - 190 185 182 208 186 187 189 179 183 191 179
  - mean 187.18
  - SD 8.02

# STATISTICAL PROCESS CONTROL

The statistical process control allows the analysis of the current trend of the production, in order to detect possible deviations from the desired target, independently on the deviation of the single object.

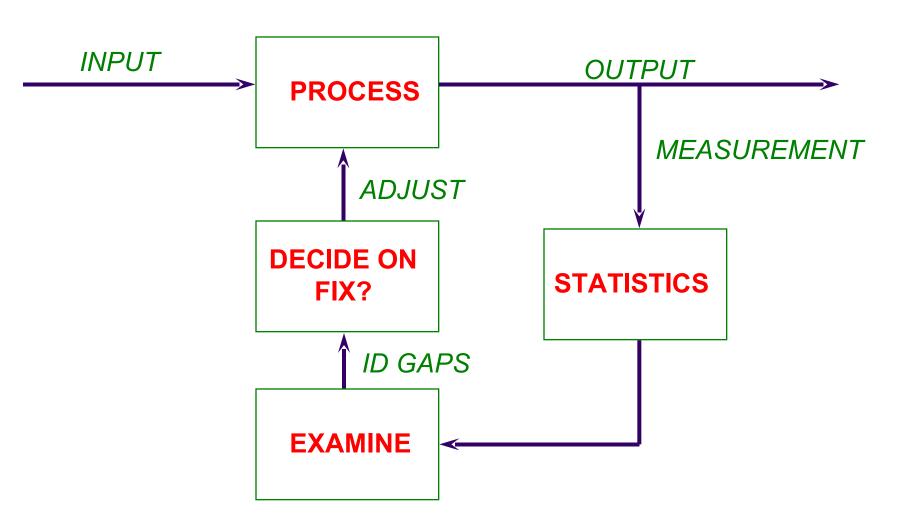
## WHAT IS SPC?



It is important to note that the **SPC** is not the cure for Quality and Production problems. SPC will only help leading to the discovery of problems and identifying the type and degree of corrective action required.

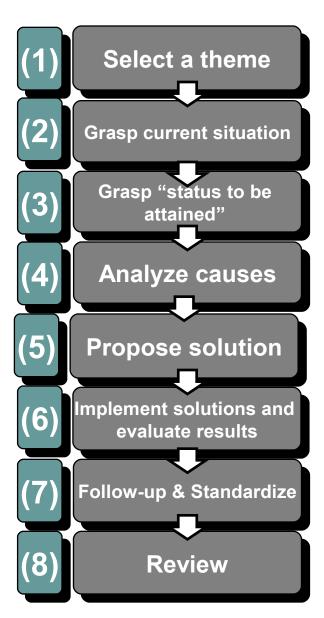
# **CONTROL LOOP**













# Seven QC Tools





## QC tools

QC tools (7 QC Tools, New 7 QC Tools) used in solving (or improving) various types of problems that occur in workshops.

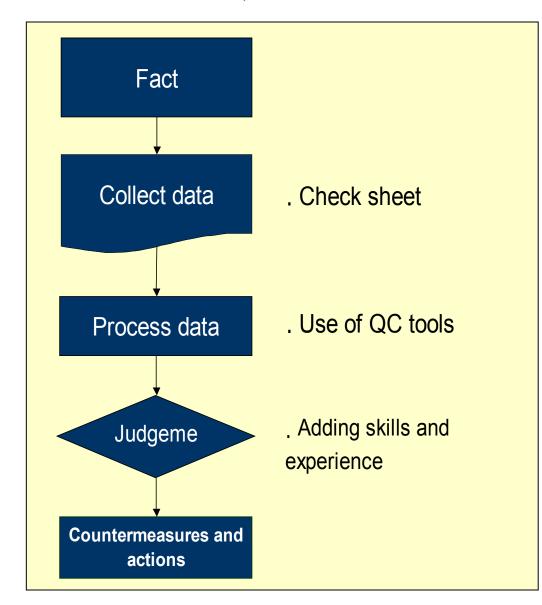
Whether in identifying causes of problems or in working out their countermeasures, effective use of QC techniques can produce good results quickly and efficiently.

It is important to get used to the use of 7 QC Tools. You are encouraged to collect actual data and practice using them.





## **Use of QC tools**



In QC-style problem-solving activity facts are grasped based on data and analyzed scientifically. Judgments are made based on facts to take concrete actions

In a situation where several factors exert influence in a complex manner, QC tools are indispensable to correctly grasp cause-and-effect relationships in order to arrive at objective judgments

# **Benefits of using QC tools**



Tool	Cat egor i es	Check sheet	Gr aphs	Pareto di agram	Scatter di agram	Hi st ogram	Control chart	Cause and effect diagram	Affinity chart	Li nkage chart	Syst em di agram	Matrix di agram	PDPC	Arrow di ragram	Flow charts	Brain storming	Brain writing
Select a theme	0		0	0				0	0	0	0	0				0	0
Shape a vision	0							0			0				0	0	0
Assess the situation	0	0	0	0	0	0	0	0	0	0	0			0	0	0	0
Anal yze causes	0	0	0	0	0	0	0	0	0	0	0					0	0
Devi se sol ut i ons	0							0		0	0	0	0		0	0	0
Implement and evaluate resu	0	0	0	0	0	0		0			0		0	0			
Follow-up	0	0					0							0	0	0	0
Revi ew	0							0	0	0						0	0

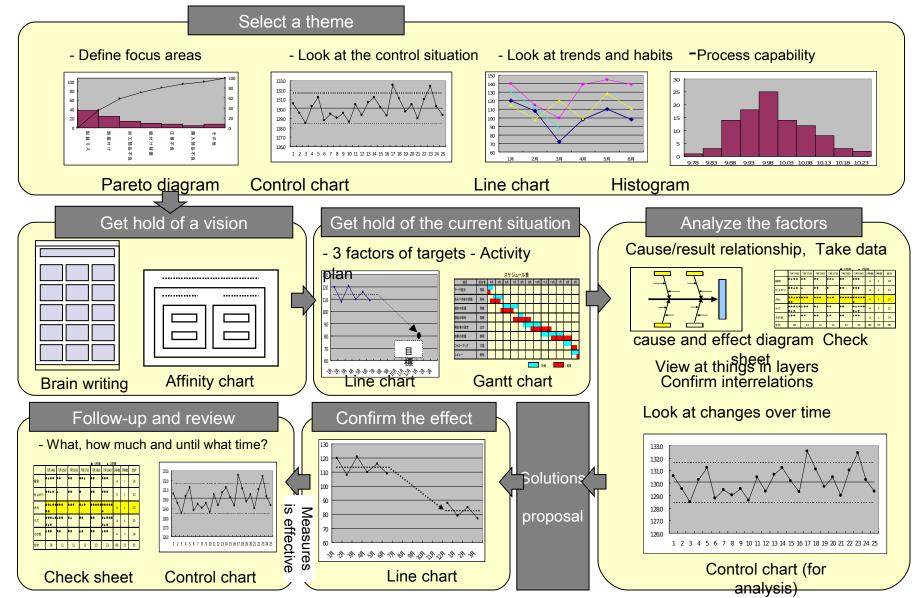
# Benefits of using QC tools



- 2. The situation can be grasped correctly, rather than based on experience or intuition
- 3. Objective judgment can be made
- 4. The overall picture can be grasped
- 5. Problem points and shortcomings become clear so that action can be taken
- 6. Problems can be shared

# **Problem solving and QC tools**





## Seven QC Tools



Stratification

Basic processing performed when collecting data

Pareto Diagram
 To identify the current status and issues

Cause and Effect Diagram
 To identify the cause and effect relationship

Histogram
 To see the distribution of data

Scatter Diagram To identify the relationship between two things

Check Sheet
 To record data collection

Control Chart
 To find anomalies and identify the current status

Graph / Flow Charts
 To find anomalies and identify the current status

## **New Seven QC Tools**



Affinity Chart

Grasp current situation and problems

Linkage Chart

Sort out relationships in the situation

System chart

- Systematic sorting of the situation
- Matrix diagram Grasp a relationship between two matters
- PDPC methods Risk management based on forecasting
- Arrow diagram
   Plan progress
- Matrix data analysis
   Correlation analysis



### **Stratification**

Stratification means to "divide the whole into smaller portions according to certain criteria." In case of quality control, stratification generally means to divide data into several groups according to common factors or tendencies (e.g., type of defect and cause of defect).

Dividing into groups "fosters understanding of a situation." This represents the basic principle of quality control.



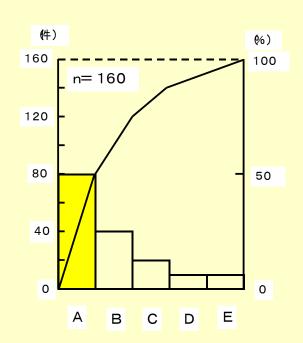


Item	Method of Stratification
Elapse of time	Hour, a.m., p.m., immediately after start of work, shift, daytime, nighttime, day, week, month
Variations among workers	Worker, age, male, female, years of experience, shift, team, newly employed, experienced worker
Variations among work methods	Processing method, work method, working conditions (temperature, pressure, and speed), temperature
Variations among measurement/ inspection methods	Measurement tool, person performing measurement, method of measurement, inspector, sampling, place of inspection



# Pareto Diagram

A Pareto diagram is a combination of bar and line graphs of accumulated data, where data associated with a problem (e.g., a defect found, mechanical failure, or a complaint from a customer) are divided into smaller groups by cause or by phenomenon and sorted, for example, by the number of occurrences or the amount of money involved. (The name "Pareto" came from an Italian mathematician who created the diagram.)



#### When is it used and what results will be obtained?

**QPSI** 

Which is the most serious problem among many problems? It is mainly used to prioritize action.

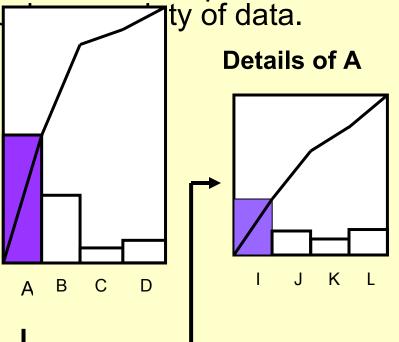
Usage	Results
<ul> <li>•Used to identify a problem.</li> <li>•Used to identify the cause of a problem.</li> <li>•Used to review the effects of an action to be taken.</li> <li>•Used to prioritize actions.</li> <li>[Used during phases to monitor the situation, analyze causes, and review effectiveness of an action.]</li> </ul>	<ul> <li>Allows clarification of important tasks.</li> <li>Allows identification of a starting point (which task to start with).</li> <li>Allows projection of the effects of a measure to be taken.</li> </ul>

# **Example usage of Pareto Diagram**



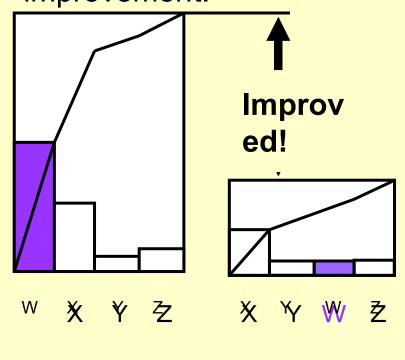
(1) Assessment using Pareto diagram (prioritization)

 To identify a course of action to be emphasized



(2) Confirmation of Effect (Comparison)

Frequently used to check the effect of an improvement.





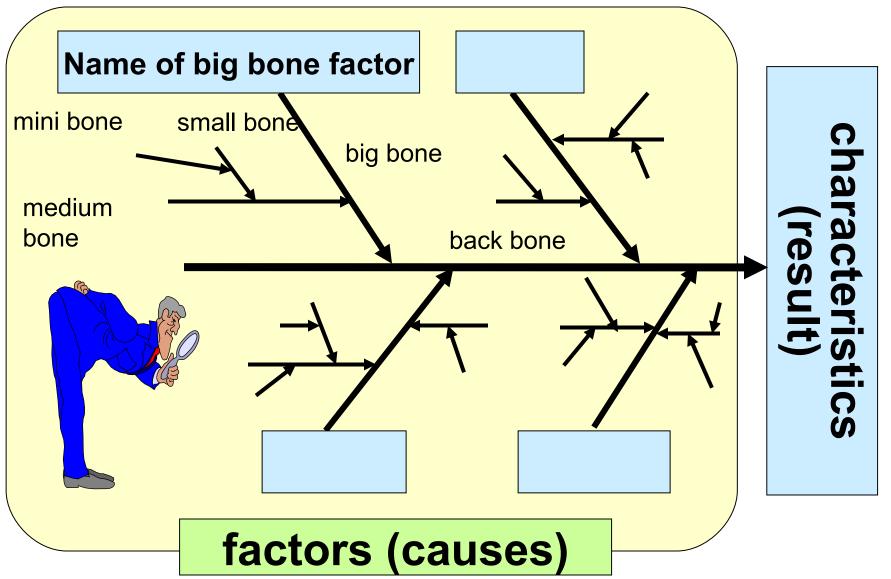
# Cause and Effect Diagram

A cause and effect diagram is "a fish-bone diagram that presents a systematic representation of the relationship between the effect (result) and affecting factors (causes).

Solving a problem in a scientific manner requires clarification of a cause and effect relationship, where the effect (e.g., the result of work) varies according to factors (e.g., facilities and machines used, method of work, workers, and materials and parts used). To obtain a good work result, we must identify the effects of various factors and develop measures to improve the result accordingly.



# **Cause and Effect Diagram**



#### When is it used and what results will be obtained?

**QPSI** 

A cause and effect diagram is mainly used to study the cause of a certain matter. As mentioned above, the use of a cause and effect diagram allows clarification of a causal relation for efficient problemsolving. It is also effective in assessing measures developed and can be applied to other fields according to your needs.

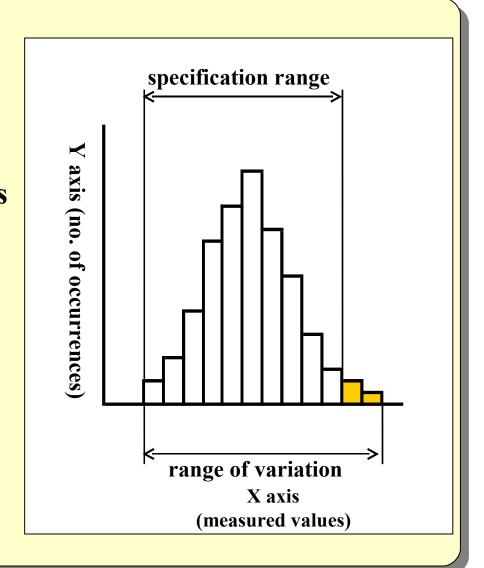
Usage	Results
<ul> <li>Used when clarifying a cause and effect relationship.</li> <li>[Used during a phase to analyzecauses.]</li> </ul>	picture of causal relation. (A
<ul> <li>•Used to develop</li> <li>•countermeasures.</li> <li>•[Used during a phase to plan countermeasures.]</li> </ul>	<ul> <li>Can clarify the cause and effect relationship.</li> <li>Can list up all causes to identify important causes.</li> <li>Can determine the direction of action (countermeasure).</li> </ul>



# **Histogram**

Articles produced with the same conditions may vary in terms of quality characteristics.

A histogram is used to judge whether such variations are normal or abnormal. First, the range of data variations are divided into several sections with a given interval, and the number of data in each section is counted to produce a frequency table. Graphical representation of this table is a histogram



# When is it used and what results will be obtained?



A histogram is mainly used to analyze a process by examining the location of the mean value in the graph or degree of variations, to find a problem point that needs to be improved. Its other applications are listed in the table

Usage	Results
[Used during phases to monitor the situation, analyze causes, and review effectiveness of an action.]	•
Used to assess the actual conditions.  Used to analyze a process to identify a problem point that needs to be improved by finding the location of the mean value or degree of variations in the graph.  Used to examine that the target quality is maintained throughout the process.	Can find out the scope of a defect by inserting standard values.  Can identify the condition of distribution (e.g., whether there is an isolated, extreme value).

## Histogram--Example No. 1



Data sheet of lengths of cut steel wire [Specification: 255±5cm] (n=100)

Nº	1	2	3	4	5	6	7	8	9	10
1	255	259	257	254	253	254	253	257	258	252
2	253	256	255	255	256	255	257	255	256	258
3	257	255	256	251	255	253	255	256	254	256
4	257	255	257	254	254	260	258	253	260	255
5	255	252	255	253	253	258	253	259	255	257
6	253	257	258	256	253	254	255	254	257	253
7	255	254	<b>253</b>	255	257	252	254	256	255	255
8	254	254	254	254	255	255	257	255	253	254
9	258	256	<b>253</b>	256	255	254	255	256	256	256
10	256	254	255	257	254	254	259	253	258	254
S	253	252	253	251	253	252	253	253	253	252
L	258	259	258	257	257	260	259	259	260	258

(Unit;cm)

#### **Histogram--Example No.2**



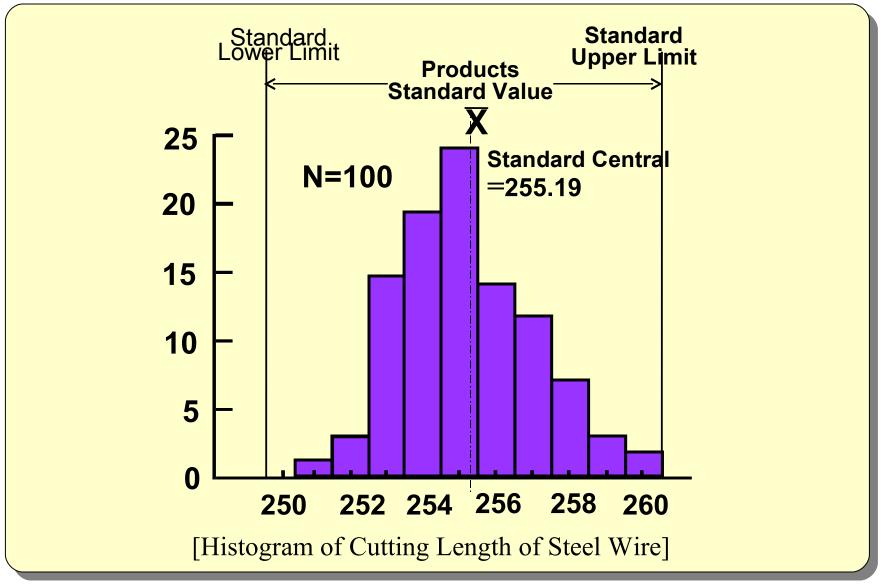
#### (Frequency Distribution Table Cutting Length of Steel Wire)

(Standard: 255± 5cm)

Nº	Section	Central Valee of Each Section	Frequency Marking	No. of Occurrences
1	250.5- 251.5	251	/	1
2	251.5- 252.5	252	///	3
3	252.5- 253.5	253	HH HH HH	15
4	253.5- 254.5	254	HH HH HH ////	19
5	254.5- 255.5	255	HL HL HL HL III	24
6	255.5- 256.5	256	HH HH ////	14
7	256.5- 257.5	257	XXX XXX //	12
8	257.5- 258.5	258	<i>}</i> }}_//	7
9	258.5- 259.5	259	///	3
10	259.5- 260.5	260	//	2
	Total	_		100

### Histogram--Example No.3





# Interpretation of Data Depicted in Histogram



Name	Description	Example	Cause
General Shape	A peak in the center, gradually declining in both directions. Almost symmetric.		A so-called "normal distribution." Means that this particular process is stable.
Trailing Type Type e	The average value (peak) is off-centered. The shape of distribution shows a relatively steep incline on one side and a moderate slope on the other. Asymmetric.		Possible causes include the standard value inserted off the center or the component of an impurity close to 0 (zero). The stability of the process is the same as that described for the General Shape.

Name	Description	Example	Cause
Twin-peak Shape	Less number of data around the center of distribution. Two peaks, one on each side.		This shape indicates the overlapping of two different distributions, when there is a variation between two machines or two workers performing the same task, often caused by one of them doing the task in a wrong way.
Plateau Shape	Small variations in the number of data around the center of distribution, forming a plateau.		Caused by the same reason described above, but with less variation.

Name	Description	Example	Cause
Precipitous Shape	The average value is extremely off-centered, showing a steep decline on one side and a moderate slope on the other. Asymmetric.	Distribution where defects seem to be excluded.	A portion of distribution depicted by dashed lines in the diagram has been removed for some reason. For example, when defective products are found during an inspection before shipping and removed from the lot, the results of an acceptance inspection performed on that lot by the customer will show this shape of distribution.

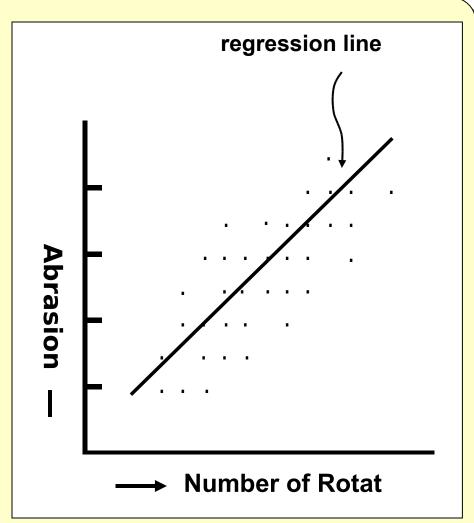
Name	Description	Example	Cause
Isolated Island Shape	The otherwise normal histogram shows an "isolated island" either on the right or left side.		This shape appears when a small amount of data from a different distribution has been accidentally included. It will be necessary to examine the data history to find anomalies in the process, errors in measurement, or the inclusion of data from another process.
Gapped Teeth Shape (or Teeth of Comb Shape)	The every other section (vertical bar) shows the number of data smaller than the one next to it, forming a gapped-teeth or teeth-of-a-comb shape.		It will be necessary to check if the width of each section has been determined by multiplying the unit (scale) of measurement with an integer, or if the person who performed the measurement has read the scale in a certain deviant manner.

# **Scatter Diagram**



A scatter diagram is used to "examine the relationship between the two, paired, interrelated data types," such as "height and weight of a person."

A scatter diagram provides a means to find whether or not these two data types are interrelated. It is also used to determine how closely they are related to identify a problem point that should be controlled or improved.



#### When is it used and what results will be obtained?

**QPSI** 

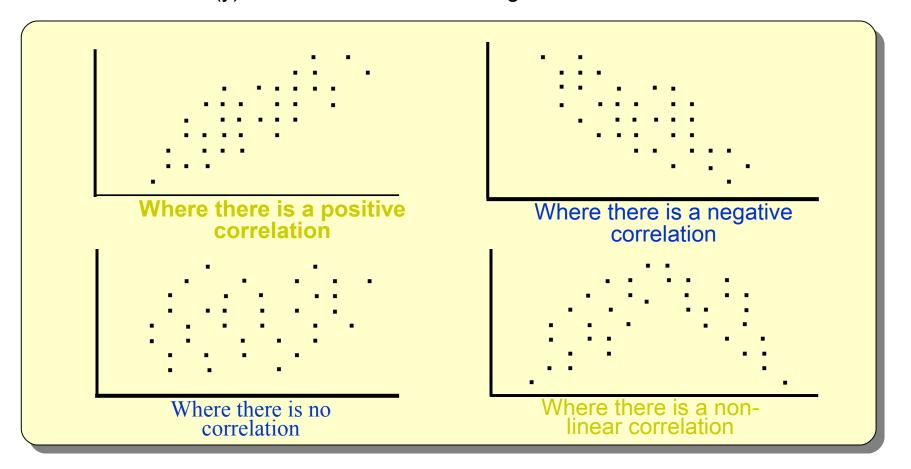
#### Used to a assess relationship between 2 data matters

Usage	Results
[Used during phases to monitor the situation, analyze causes and review effectiveness of an action.]	Can identify cause and effect relation.
Used to identify a relationship between two matters.	(Can understand the
Used to identify a relationship between two matters and establish countermeasures based on their cause and effect relation.	relationship between two results.)
Example Usage	
•Relationship between thermal treatment temperature of a steel material and its tensile strengths.	
•Relationship between visit made by a salesman and volume of sales.	
•Relationship between the number of persons visiting a department store and volume of sales	

#### **Various Forms of Scatter Diagram**



The table below shows some examples of scatter diagram's usage. If, for example, there is a relationship where "an increase in the number of rotations (x) causes an increase in abrasion (y)," there exists "positive correlation." If, on the other hand, the existence of a relationship where "an increase in the number of rotations (x) causes a decline in abrasion (y)" indicates that there is "negative correlation."





### **Check Sheet**

A check sheet is "a sheet designed in advance to allow easy collection and aggregation of data." By just entering check marks on a check sheet, data can be collected to extract necessary information, or a thorough inspection can be performed in an efficient manner, eliminating a possibility of skipping any of the required inspection items.

A check sheet is also effective in performing stratification (categorization).

# Example Usage of Check Sheet



#### A check sheet used to identify defects

Date Defect	6/10	6/11	6/12	6/13	6/14	Total
Vertical Scratch	<i>}</i>	////	<b> </b>	////	<b>}</b>	34
Scratch	//		///	///	//	11
Dent	//// /	 	////	/// //	/// ////	37

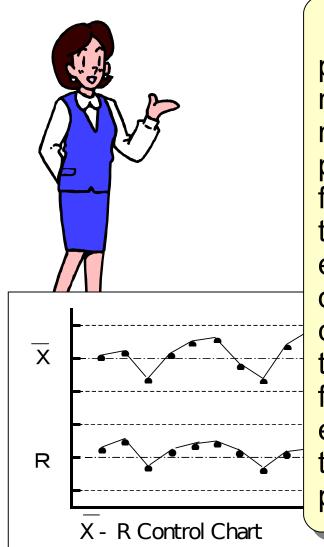
#### When is it used and what results will be obtained?



Usage	Results
<ul> <li>Used to collect data.</li> <li>Used when performing a thorough inspection</li> <li>Used to identify the actual condition of a situation.</li> <li>(Used during phases to monitor the situation, analyze causes, review effectiveness of an action, perform standardization, and implement a selected control measure.)</li> </ul>	<ul> <li>Ensures collection of required data.</li> <li>Allows a thorough inspection of all check items.</li> <li>Can understand tendencies and variations.</li> <li>Can record required data.</li> </ul>

## QPSP

#### **Control Chart**



A control chart is used to examine a process to see if it is stable or to maintain the stability of a process. This method is often used to analyze a process. To do so, a chart is created from data collected for a certain period of time, and dots plotted on the chart are examined to see how they are distributed or if they are within the established control limit. After some actions are taken to control and standardize various factors, this method is also used to examine if a process has stabilized by these actions, and if so, to keep the process stabilized.

#### When is it used and what results will be obtained?



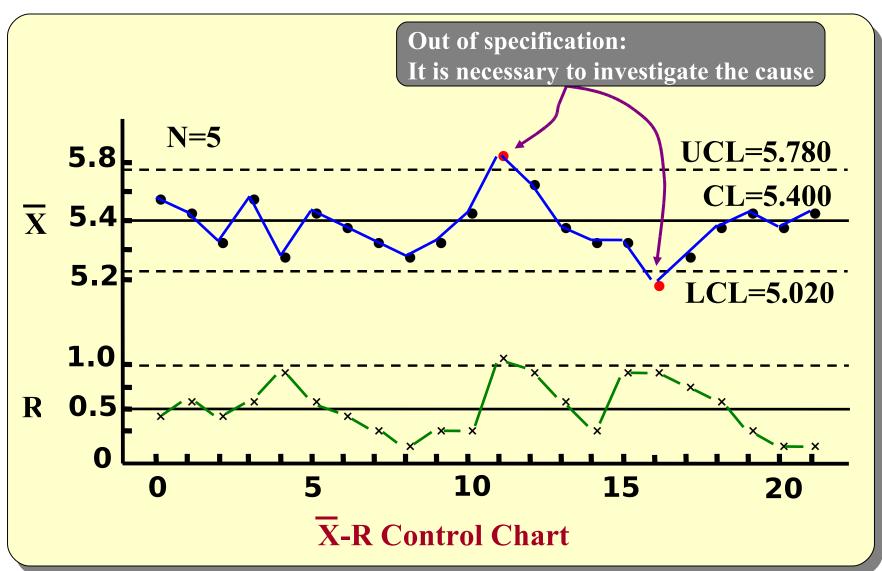
Usage	Results
[Used during phases to monitor the situation, analyze causes, review effectiveness of an action, perform Standardization and implement a selected control measure.]  Used to observe a change caused by elapse of time.	Can identify a change caused by elapse of time.  Can judge the process if it is in its normal state or there are some anomalies by examining the dots plotted on the chart.  In the example x(-)-R control chart, x(-)" represents the central value, while "R" indicates the range.

Control Chart for Managerial Purposes: Extends the line indicating the control limit used for analytical purposes to plot data obtained daily to keep a process in a good state.

<sup>\*</sup> Control Chart for Analytical Purposes: Examines a process if it is in a controlled state by collecting data for a certain period of time. If the process is not controlled, a survey is performed to identify its cause and develop countermeasures.



## **Major Application**



## Graph



A graph is "a graphical representation of data, which allows a person to understand the meaning of these data at a glance."

Unprocessed data simply represent a list of numbers, and finding certain tendencies or magnitude of situation from these numbers is difficult, sometimes resulting in an interpretational error.

A graph is a effective means to monitor or judge the situation, allowing quick and precise understanding of the current or actual situation.

A graph is a visual and summarized representation of data that need to be quickly and precisely conveyed to others.

#### When is it used and what results will be obtained?

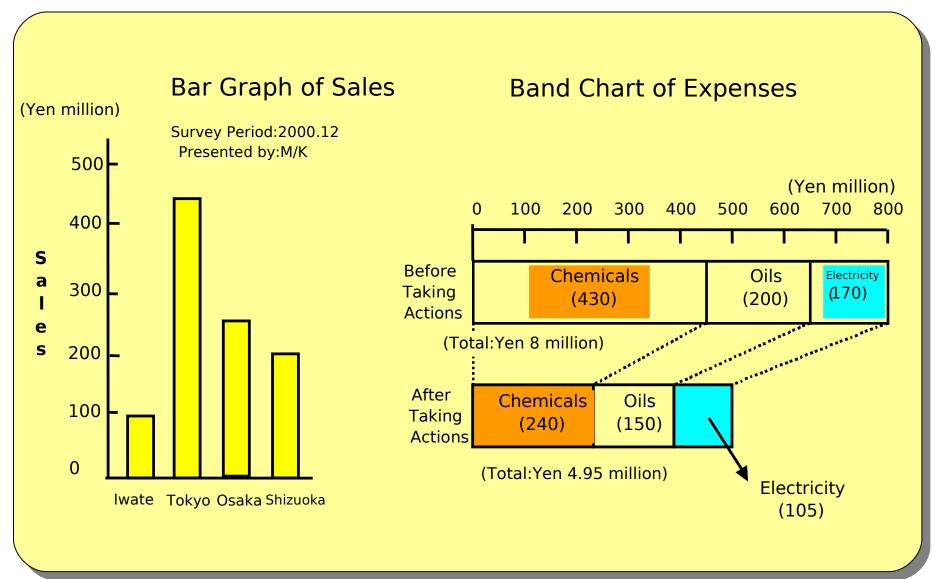
**QPSI** 

A graph, although it is listed as one of the QC tools, is commonly used in our daily life and is the most familiar means of assessing a situation.

Usage	Usage	Results
	Used to observe changes in a time-sequential order (line graph) Used to compare size (bar graph) Used to observe Ratios (pie graph, column graph)	A graphs is the most frequently used tool among QC 7 tools. Can recognize changes in a timesequential order, ratios, and size.

#### **Example usage of Graph**







## **Control Charts**





## The History of Control Charts

- Developed in the 1920's
- Dr. Walter Shewhart, then an employee of Bell Laboratories developed the control chart to separate the special causes of variation from the common causes of variation.

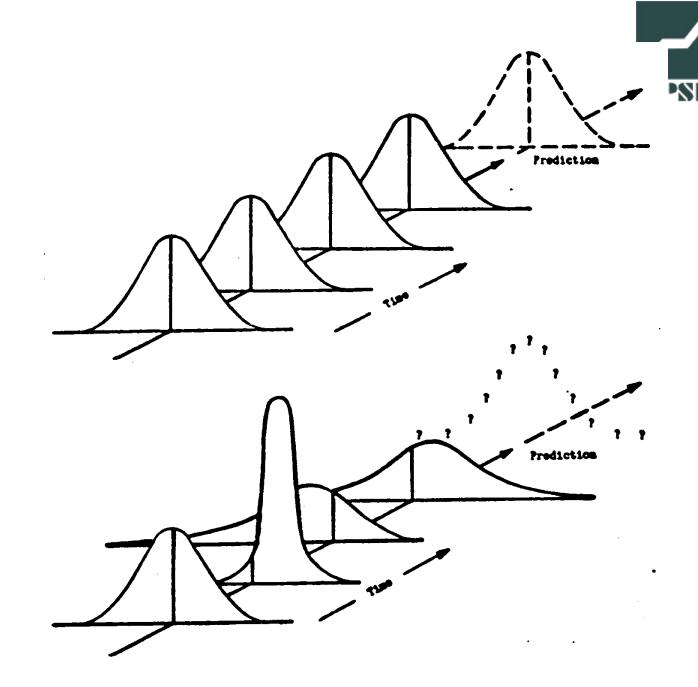


#### **Statistical Process Control (SPC)**

- A methodology for monitoring a process to identify special causes of variation and signal the need to take corrective action when appropriate
- SPC relies on control charts

Common Causes

Special Causes





#### COMMON CAUSE

- RANDOM VARIATION
- SUM OF MANY SMALL VARIANCES
- SYSTEM-RELATED
- 80% OF PROCESS VARIATION
- RESPONSIBILITY OF MANAGEMENT
- WRONGLY ATTRIBUTED TO LINE EMPLOYEES

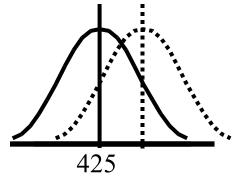
#### SPECIAL CAUSES



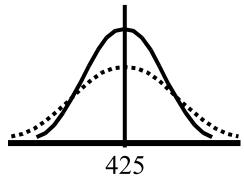
- ASSIGNABLE
- 20% OF PROCESS VARIATION
- IDENTIFIABLE TO SPECIFIC CONDITIONS
- OVERCOME BY REMOVAL, TRAINING, EXPERIENCE and/or COACHING

#### (2) Assignable causes variation:

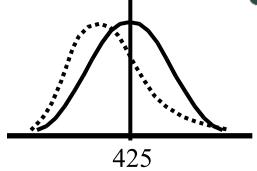




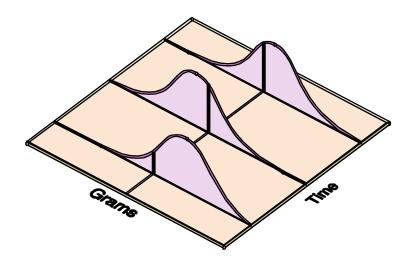




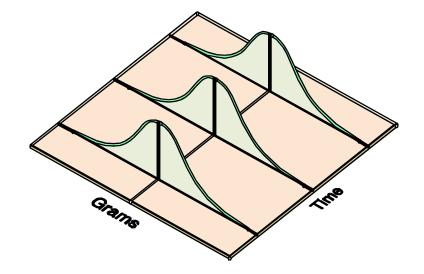
(b) Spread



(c) Shape

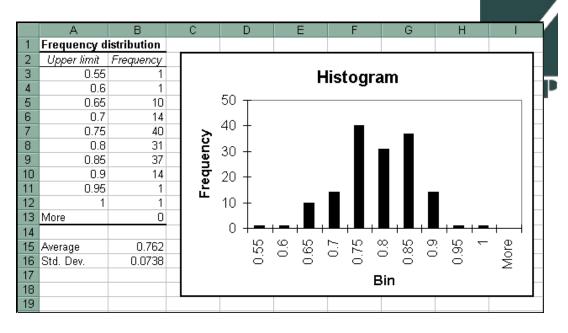


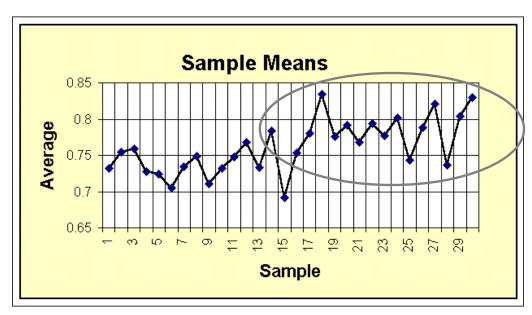
Out of control (assignable causes present)



In control (no assignable causes)

Histograms do not take into account changes over time.





Control charts can tell us when a process changes



## **Control Chart Applications**

- Establish state of statistical control
- Monitor a process and signal when it goes out of control
- Determine process capability



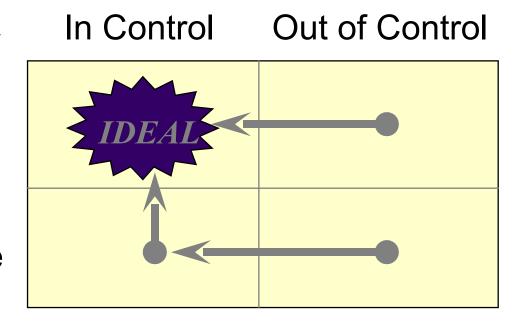
#### **Capability Versus Control**

#### **Control**

Capability

Capable

Not Capable





#### **Commonly Used Control Charts**

- Variables data
  - x-bar and R-charts
  - x-bar and s-charts
  - Charts for individuals (x-charts)
- Attribute data
  - For "defectives" (p-chart, np-chart)
  - For "defects" (c-chart, u-chart)



## **Developing Control Charts**

#### 1. Prepare

- Choose measurement
- Determine how to collect data, sample size, and frequency of sampling
- Set up an initial control chart

#### 2. Collect Data

- Record data
- Calculate appropriate statistics
- Plot statistics on chart



## **Next Steps**

- 1. Determine trial control limits
  - Center line (process average)
  - Compute UCL, LCL
- 2. Analyze and interpret results
  - Determine if in control
  - Eliminate out-of-control points
  - Recompute control limits as necessary

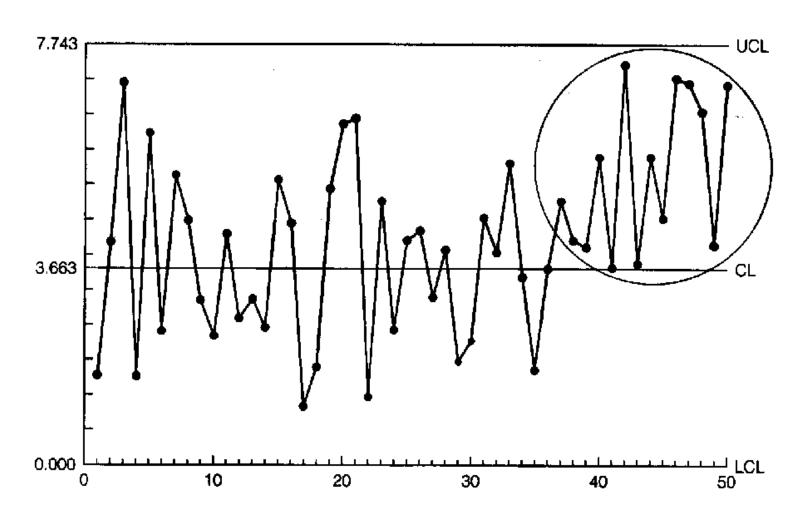


#### **Typical Out-of-Control Patterns**

- Point outside control limits
- Sudden shift in process average
- Cycles
- Trends
- Hugging the center line
- Hugging the control limits
- Instability

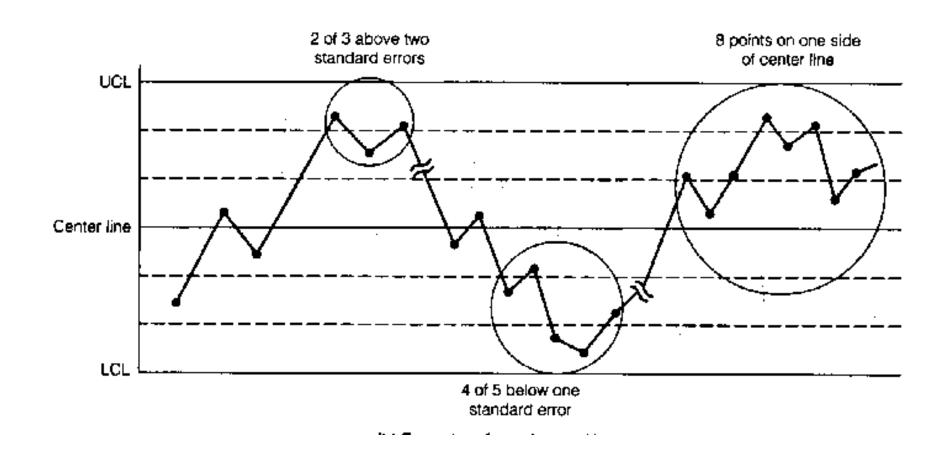


## **Shift in Process Average**



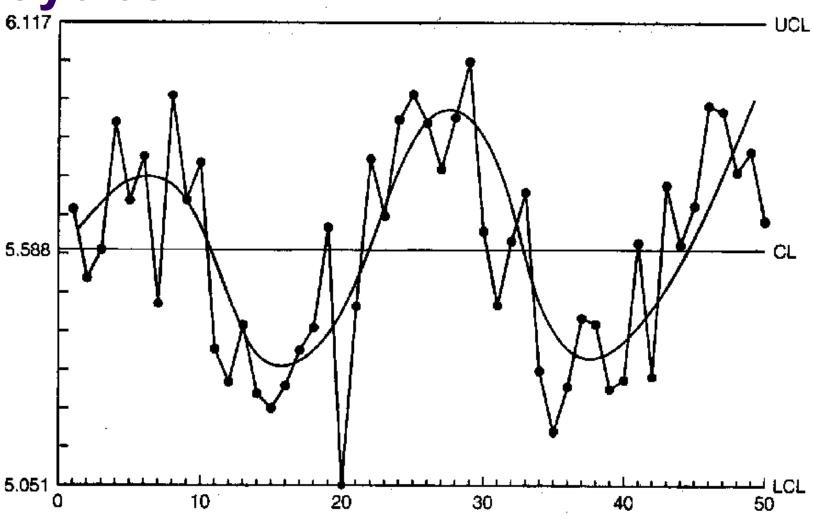


## **Identifying Potential Shifts**



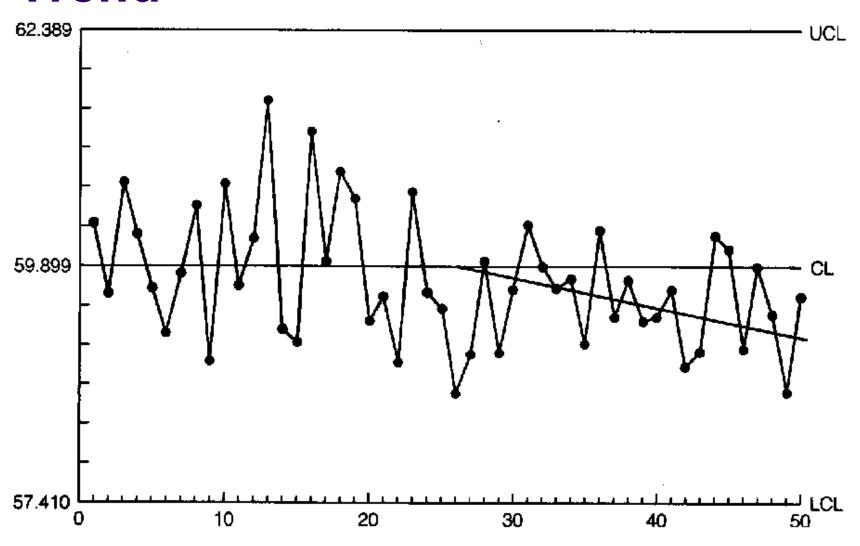


Cycles 6.117





## **Trend**





## **Final Steps**

- 1. Use as a problem-solving tool
  - Continue to collect and plot data
  - Take corrective action when necessary
- 2. Compute process capability

## **Process Capability Calculations**

#### **CALCULATION WORK SHEET**

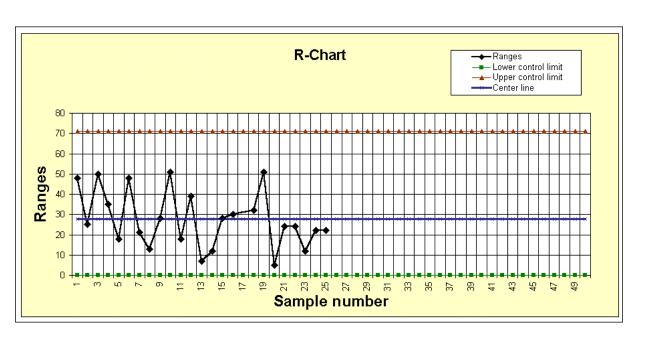
	CONTROL LIMITS				
SUBGROUPS INCLUDED ALL			# 17 REMOVED		
	Ā - ΣΑ - 676 25	~ 27	<u>443</u> -	27.6	
	$\overline{X} = \frac{\Sigma \overline{X}}{k} = \frac{1221}{25}$	- 4 <b>3</b> . 8	<u>1127</u> -	<del>4</del> 7.0	
	X' (MIDSPEC. OR STD.)	- 50	_	50	
	A2R -1-023 x 27	_ 27.6	1.023 ×27.6 -	28.2	
	UCL <sub>국</sub> = X + A2주	- 76.4	-	75.2	
	LCL <sub>x</sub> = X - A₂Ā	- 21. 2.	-	18.8	
	UCL <sub>B</sub> - D4A -2.574 x 2	27 - 69.5	2.574 × 27.6=	71.0	

LIMITS FOR INDIV	IDUALS		
COMPARE WITH SPECIFICATION OR TOLERANCE LIMITS			
₹	- 47.0		
3 R = 1.772 × 27	1.6 - 41.9		
$UL_{x} = \overline{X} + \frac{3}{d_{x}}\overline{R}$	- 95.9		
$\coprod_{x} = \overline{X} - \frac{3}{d_2} \overline{R}$	<u> </u>		
υs	_ 100		
LS	- 0		
US – LS	- 100		
60 - 6 A	- 97.8		

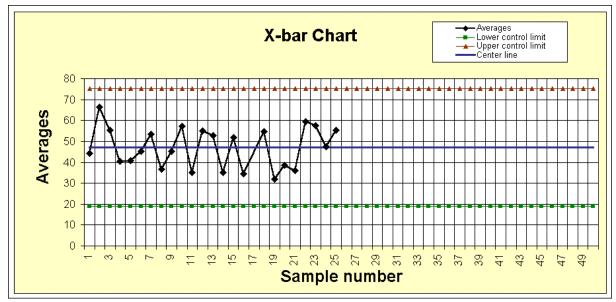
# MODIFIED CONTROL LIMITS FOR AVERAGES BASED ON SPECIFICATION LIMITS AND PROCESS CAPABILITY. APPLICABLE ONLY IF: US-LS > 6\(\sigma\). US = LS = A\_\text{A}\text{R} = \text{X} = \text{A\_\text{A}\text{R}} = \text{LS} = LS + A\_\text{R}\text{R} =

. —					
<u>n</u>	_A <sub>2</sub> _	D <sub>4</sub>	<u>d</u> e	3 d <sub>2</sub>	_ <u>_</u>
2	1 880	3,268	1.128	2.659	0.779
3		(2.574)	1.693	(1.772)	0.749
4	0.729	2.282	2.059	1.457	0.728
5	0.577	2.114	2.326	1.290	0.713
6	0.483	2.004	2.534	1.184	0.701
	2 3 4 5	2 1.880 3 0.729 4 0.729 5 0.577	2 1 880 3 268 3 1023 2 574 4 0.729 2 282 5 0.577 2.114	2 1.880 3.268 1.128 3 0.023 2.574 1.693 4 0.729 2.282 2.059 5 0.577 2.114 2.326	n         A2         D4         d2         d2           2         1.880         3.268         1.128         2.659           3         1.023         2.574         1.693         1.772           4         0.729         2.282         2.059         1.457           5         0.577         2.114         2.326         1.290

**FACTORS FOR CONTROL LIMITS** 



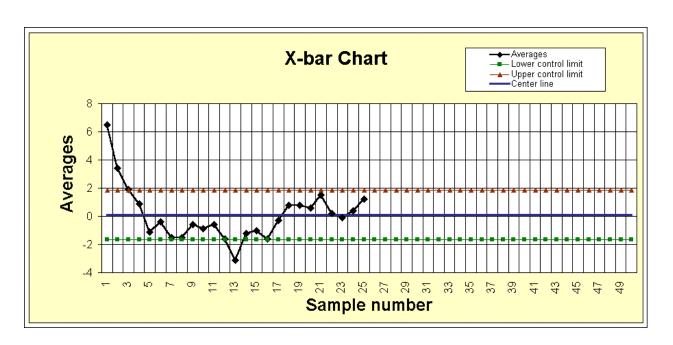




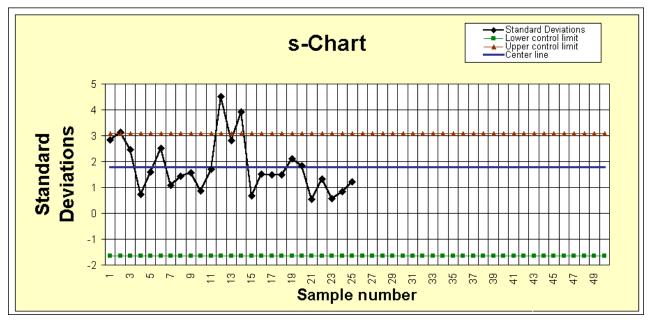


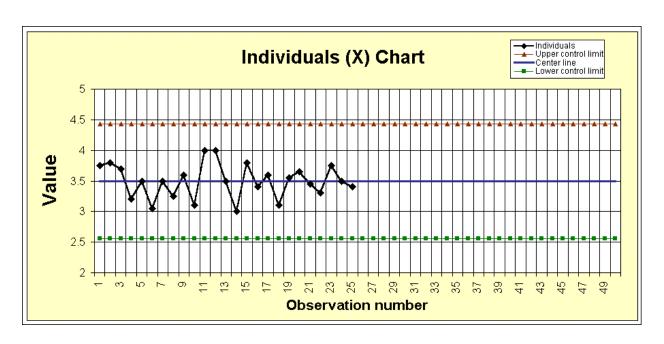
## **Special Variables Control Charts**

- x-bar and s charts
- x-chart for individuals

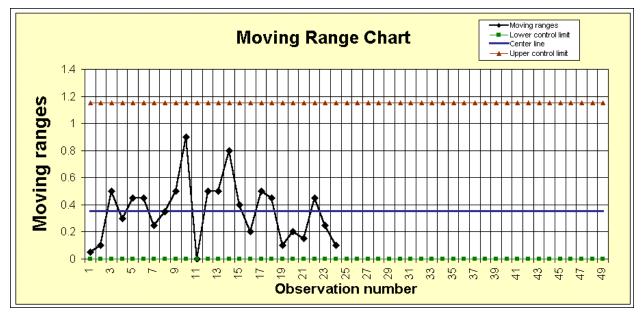








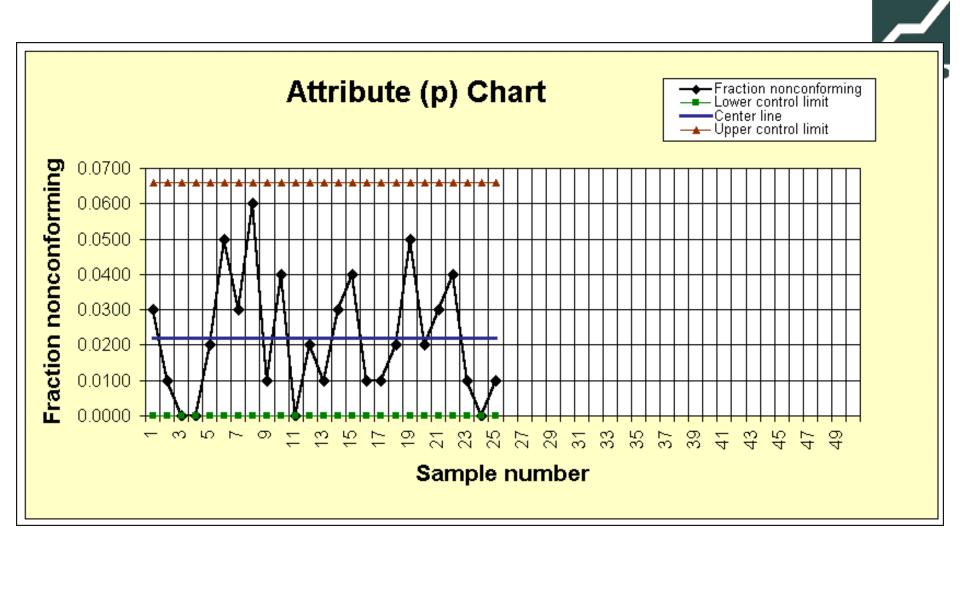


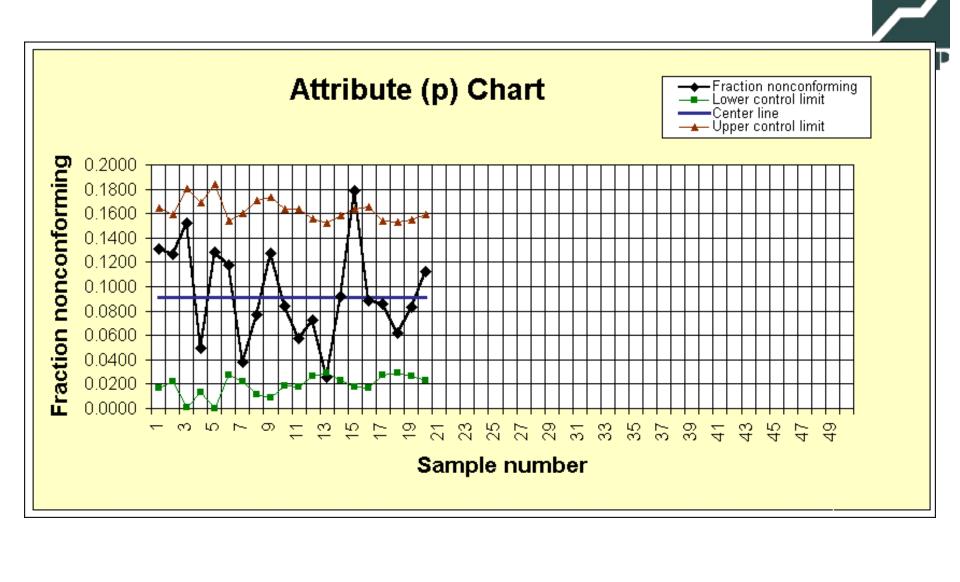




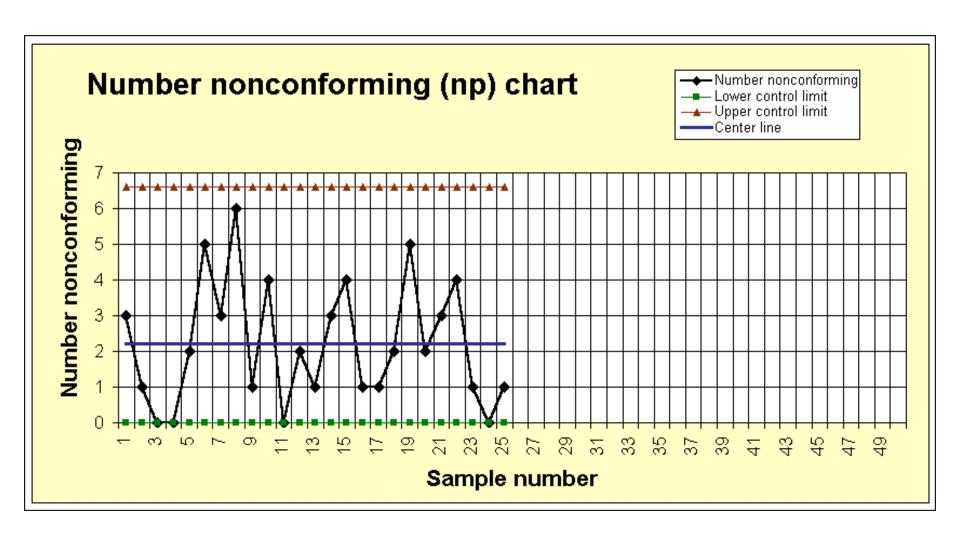
#### **Charts for Attributes**

- Fraction nonconforming (p-chart)
  - Fixed sample size
  - Variable sample size
- np-chart for number nonconforming
- Charts for defects
  - c-chart
  - u-chart

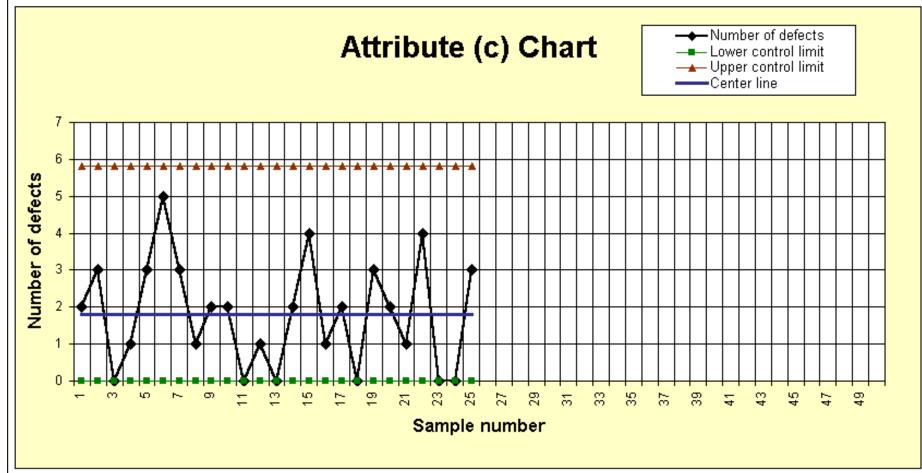




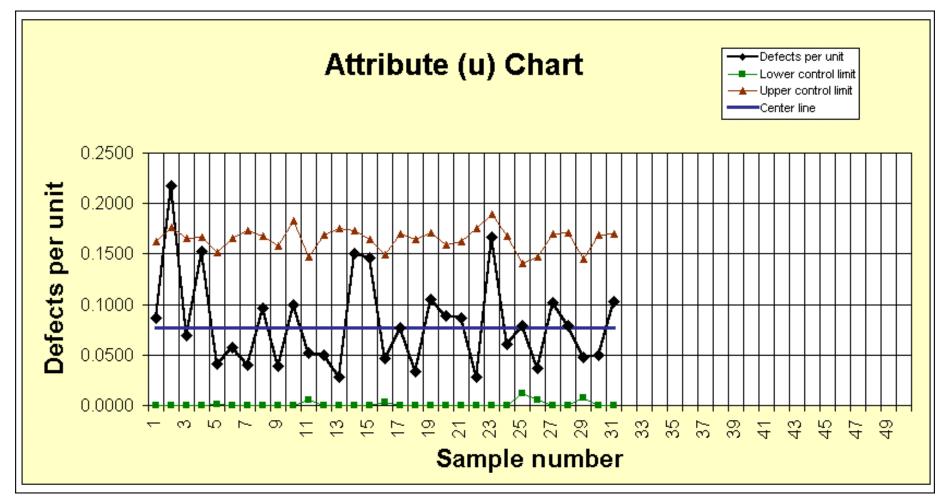






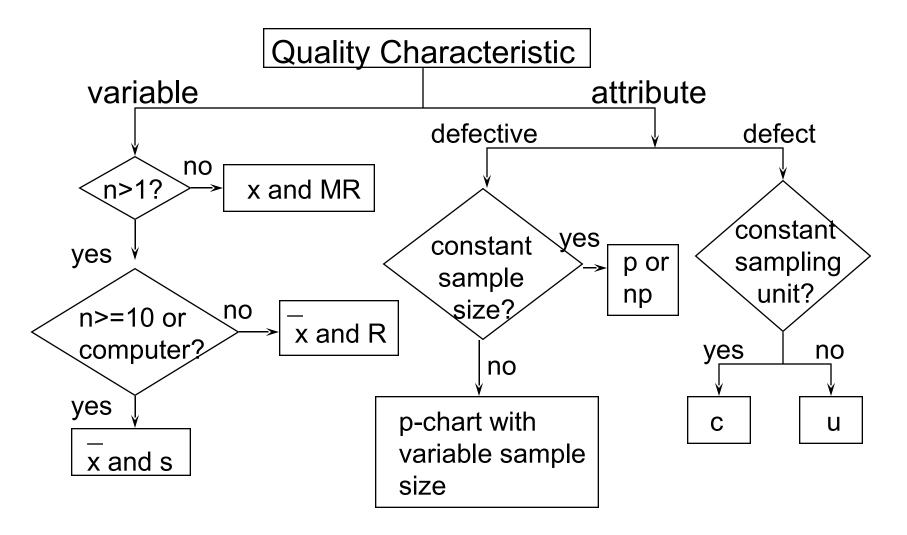








#### **Control Chart Selection**



### **Types of Shewhart Control Charts**



#### **Control Charts for Variables Data**

X and R charts: for sample averages and ranges.

 $\overline{X}$  and s charts: for sample means and standard deviations.

Md and R charts: for sample medians and ranges.

X charts: for individual measures; uses moving ranges.

#### **Control Charts for Attributes Data**

p charts: proportion of units nonconforming.

np charts: number of units nonconforming.

c charts: number of nonconformities.

u charts: number of nonconformities per unit.

#### **The Central Limit Theorem**



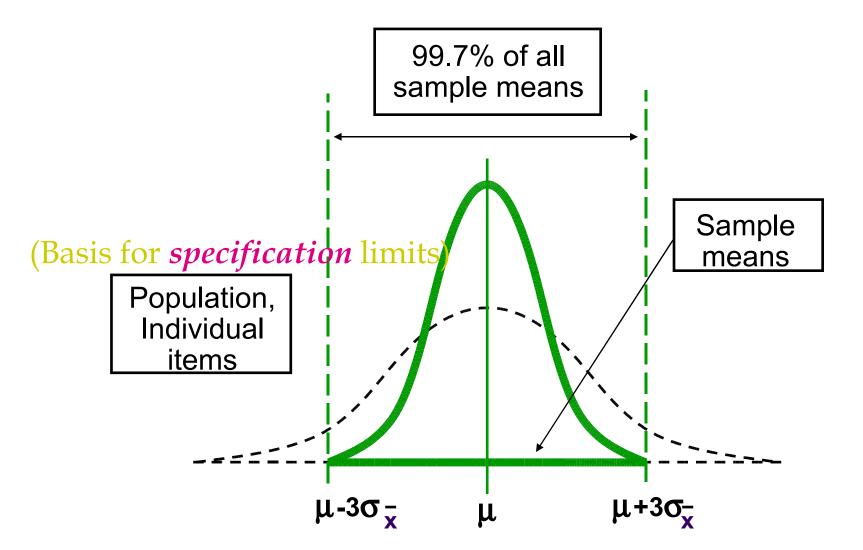
Suppose a population has a mean  $(\mu)$  and a standard deviation  $(\sigma)$ 

**The Central Limit Theorem states** 

The distribution of sample means (X) will be approximately normal. Its mean  $\bar{\bar{x}} = \mu$ , and its standard deviation  $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ 

#### **Central Limit Theorem Illustrated**







- Logic Behind Control Charts
  - Consider measurement of variables data
    - We know that a sample average typically varies from the population average.
    - The problem is to determine if any variation from a specified population average is
      - Is simply random variation
      - Or is because the population average is not as specified
  - We therefore establish limits on how different we'll allow the sample average (or whatever other summary measure) to be before we conclude the specification is not being met.
- Control Limits Set via Sampling Theory



- The Good News:
  - We don't need to go back to the statistics books and tables
  - Simple-to-use tables and formulae have been developed for creating control charts
    - Formulae and tables for variables data
    - Formulae only for attributes data





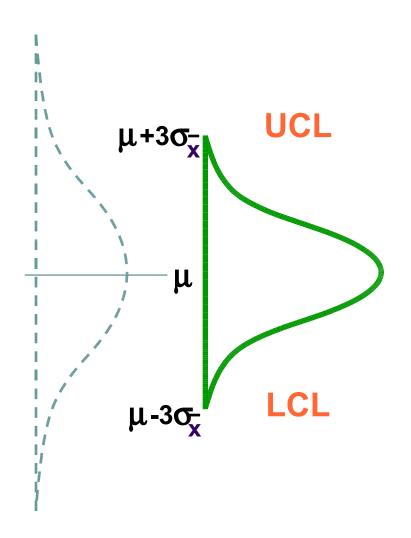
Sample (Subgroup) Size (n)	Control Limit Factor for Averages (Mean Charts) (A <sub>2</sub> )	for Ranges (Range Charts) (D <sub>4</sub> )	for Ranges (Range Charts) (D <sub>3</sub> )	Factor for Estimating Sigma (  ( (d <sub>2</sub> )
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059
5	0.577	2.115	0	2.326
6	0.483	2.004	0	2.534
7	0.419	1.924	0.076	2.704
8	0.373	1.864	0.136	2.847
9	0.337	1.816	0.184	2.970
10	0.308	1.777	0.223	3.078



- Process Overview
  - First, develop sampling plan:
    - Number of observations per sample
    - Frequency of sampling
  - Stage 1 sampling:
    - Conduct initial periodic sampling
    - Determine control limits
    - Perform calculations
    - Decide whether in control or not
  - Stage 2 sampling (only if Stage 1 is successful):
    - Continue operating with periodic sampling
    - Perform calculations
    - Decide whether in control (each sample)

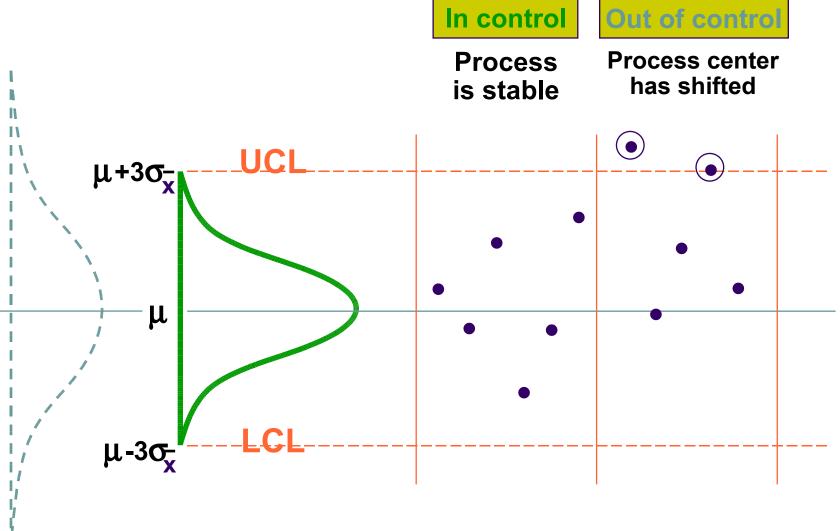


### **SPC: Control Limits**



### **SPC: Control Limits**





Values

Sum

 $\overline{\mathbf{X}}$ 

R



Select 25 small samples (in this case, n=4)

Find X and R of each sample.

The X chart is used to control the process mean.

The R chart is used to control process variation.



n	A <sub>2</sub>	$D_{\scriptscriptstyle{4}}$	$D_3$	$d_2$
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059

	Sample Number				
	1	2	3	4	25
Ñ	4	7	6	7	} _
Values	6	3	9	6	
Va	_5_	8	_8_	6	
	5	6	9	5	
Sum	20	24	32	24	<u> </u>
X	_5_	6	_8_	_6	
R	2	5	3	2	$\left\langle \right\rangle$

**Total** 150 75



n	A <sub>2</sub>	$D_{\scriptscriptstyle{4}}$	$D_3$	$d_2$
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059

X = 150 / 25 = 6
= R = 75 / 25 = 3
$A_2R = 0.729(3) = 2.2$
$UCL_X = X + A_2R = 6 + 2.2 = 8.2$
$LCL_{x}^{-} = X - A_{2}R = 6 - 2.2 = 3.8$
$UCL_R = D_4R = 2.282(3) = 6.8$

		San	nple	Nu	ımber
	1	2	3	4	25
S	4	7	6	7	}
Values	6	3	9	6	
Va	_5	8	_8_	6	
	5	6	9	5	<b>&gt;</b>
Sum	20	24	<b>32</b>	24	
X	_5_	6	_8_	_6	
R	2	5	3	2	$\langle \langle \langle \rangle \rangle$

**Total** 150 75

 $\sigma = R/d$ 

 $LCL_{R} = D_{3}R = 0(3) = 0$ 



n	A <sub>2</sub>	$D_4$	$D_3$	$d_2$
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059

$$X = 150 / 25 = 6$$

$$R = 75 / 25 = 3$$

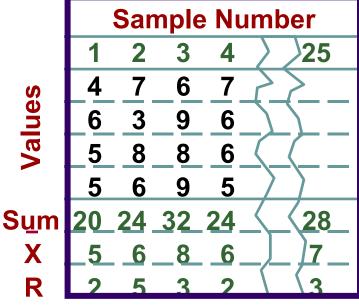
$$A_2R = 0.729(3) = 2.2$$

$$UCL_x = X + A_2R = 6 + 2.2 = 8.2$$

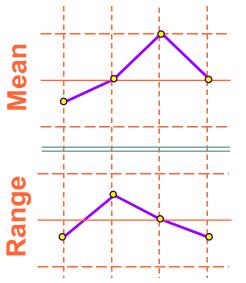
$$LCL_{X}^{-} = \overline{X} - A_{2}R = 6 - 2.2 = 3.8$$

$$UCL_{R}^{-} = D_{4}R = 2.282(3) = 6.8$$

$$LCL_{R} = D_{3}\underline{R} = 0(3) = 0$$



**Total** 150



$$UCL_{\bar{x}} = 8.2$$
  
= X = 6.0

$$LCL_{\bar{X}} = 3.8$$

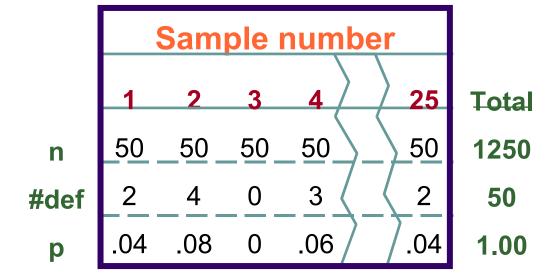
$$UCL_R = 6.8$$

$$\bar{R} = 3.0$$

$$LCL_R = 0$$



### p Chart



### p Chart

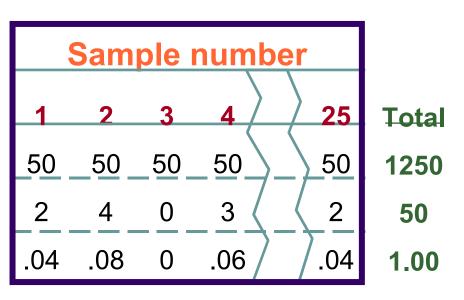


$$\bar{p} = \frac{\sum \#def}{\sum n} = 50/1250 = .04$$

$$3\sigma_{P} = 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 3\sqrt{\frac{.04(.96)}{50}}$$

$$= 0.083$$



$$UCL_{p} = \bar{p} + 3\sigma_{p}$$
  
= .04 + .083 = .123

UCL 
$$_{P} = \bar{p} - 3\sigma_{P}$$
  
= .04 - .083 = 0

### p Chart



$$\bar{p} = \frac{\sum \#def}{\sum n} = 50/1250 = .04$$

$$3\sigma_{P} = 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

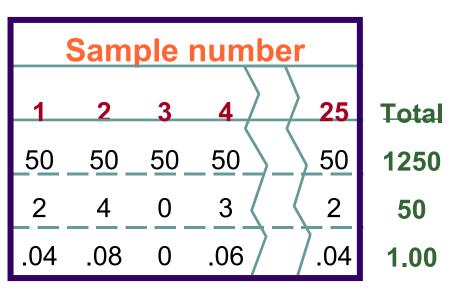
$$= 3\sqrt{\frac{.04(.96)}{50}}$$

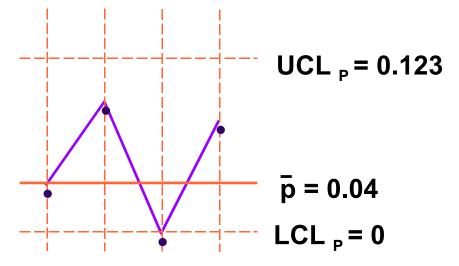
$$= 0.083$$

UCL 
$$_{P} = \bar{p} + 3\sigma_{_{P}}$$
  
= .04 + .083 = .123

UCL 
$$_{P} = \bar{p} - 3\sigma_{P}$$
  
= .04 - .083 = 0

n #def p





can't be negative

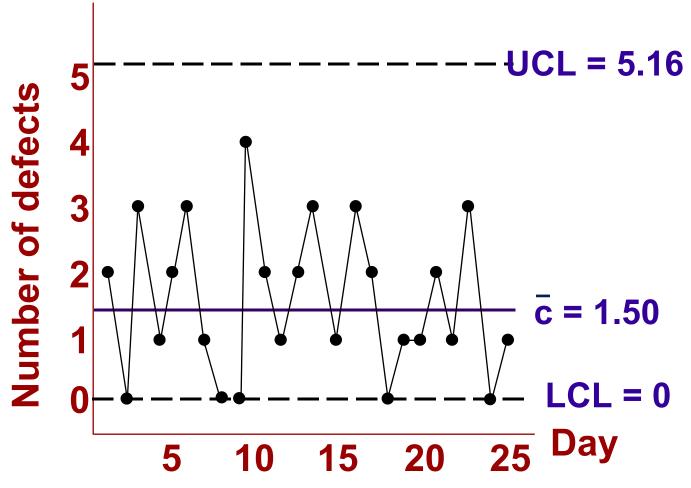


## **Hotel Suite Inspection - Defects Discovered**

Day	Defects	Day	Defects	Day	Defects
1	2	10	4	19	1
2	0	11	2	20	1
3	3	12	1	21	2
4	1	13	2	22	1
5	2	14	3	23	0
6	3	15	1	24	3
7	1	16	3	25	0
8	0	17	2	<b>26</b>	1
9	0	18	0		_
				Tot	tal 39

# c Chart for Hotel Suite Inspection





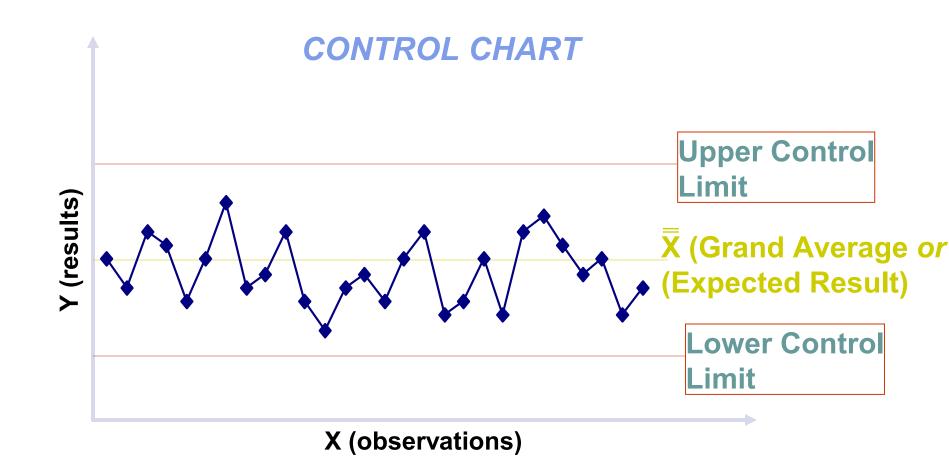
## CONTROL CHARTS WHY INSPECTION DOESN'T WORK



- IN ORDER TO CONSISTENTLY SHIP QUALITY PRODUCT TO THE CUSTOMER YOU HAVE TO MONITOR THE PROCESS NOT THE PRODUCT
- INSPECTION (if you're lucky) FINDS DEFECTS AFTER THE FACT
- THIS RESULTS IN C.O.P.Q. COSTS THAT COULD HAVE BEEN DETECTED OR AVOIDED MUCH EARLIER IN THE PROCESS

## CONTROL CHARTS THE BASICS

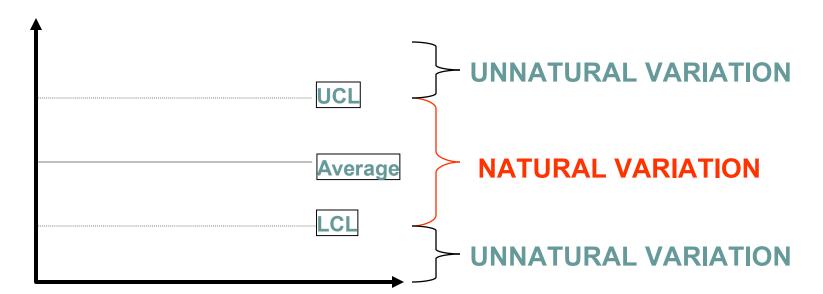




## **CONTROL CHARTS**VARIATION



- CONTROL CHARTS DISTINGUISHES BETWEEN:
  - NATURAL VARIATION (COMMON CAUSE)
  - UNNATURAL VARIATION (SPECIAL CAUSE)



# CONTROL CHARTS X<sub>BAR</sub> - R CHART STEPS (1)



- DETERMINE SAMPLE SIZE (n=2-6)
- DETERMINE FREQUENCY OF SAMPLING
- COLLECT 20-25 DATA SETS
- AVERAGE EACH SAMPLE (X-bar)
- RANGE FOR EACH SAMPLE (R)
- AVERAGE OF SAMPLE AVERAGES =
  - X-double bar
- AVERAGE SAMPLE RANGES =
  - R-bar

# CONTROL CHARTS X<sub>BAR</sub> - R CHART STEPS (2)



#### • X<sub>BAR</sub> CONTROL LIMITS:

$$UCL = X_{DBAR} + (A_2)(R_{BAR})$$

$$LCL = X_{DBAR} - (A_2)(R_{BAR})$$

#### R CONTROL LIMITS:

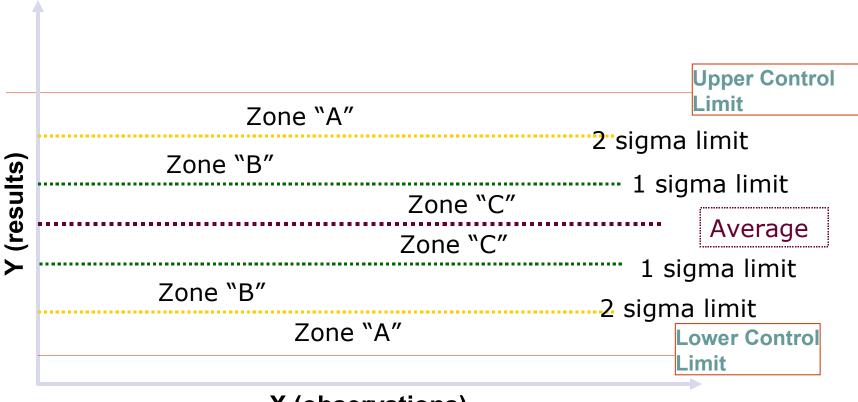
$$- UCL = (D_4)(R_{BAR})$$

$$LCL = (D_3)(R_{BAR})$$



### Determining if your control Chart is "Out of Control"

Control Chart



X (observations)



### Tests for Assignable (special) causes

Test 1	One point beyond 3 sigma
Test 2	Nine points in a row on one side of the centerline
Test 3	Six points in a row steadily increasing or decreasing
Test 4	Fourteen points in a row alternating up and down
Test 5	Two out of three points in a row beyond 2 sigma
Test 6	Four out of five points in a row beyond 1 sigma
Test 7	Fifteen points in a row within I sigma of the
cente	

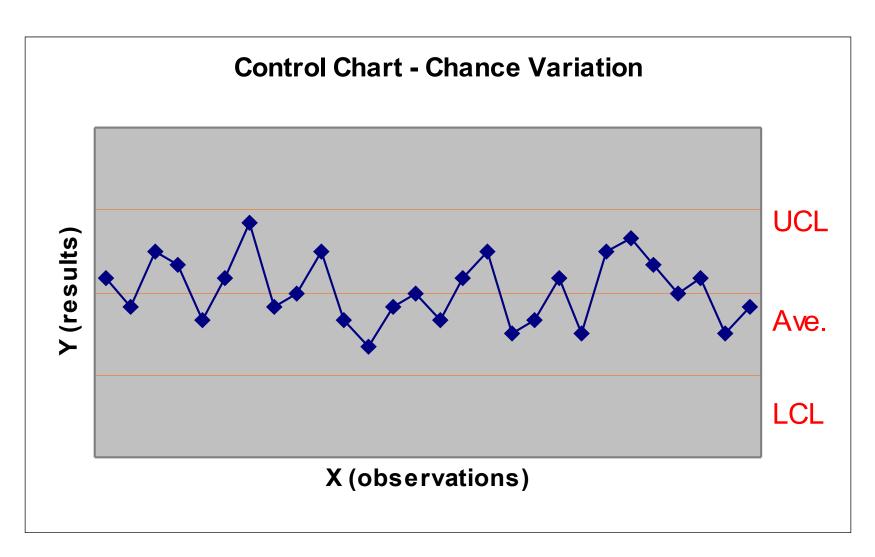
Test 8 Eight points in a row on both sides of the centerline, all beyond 1 sigma

## CONTROL CHARTS INTERPRETATION



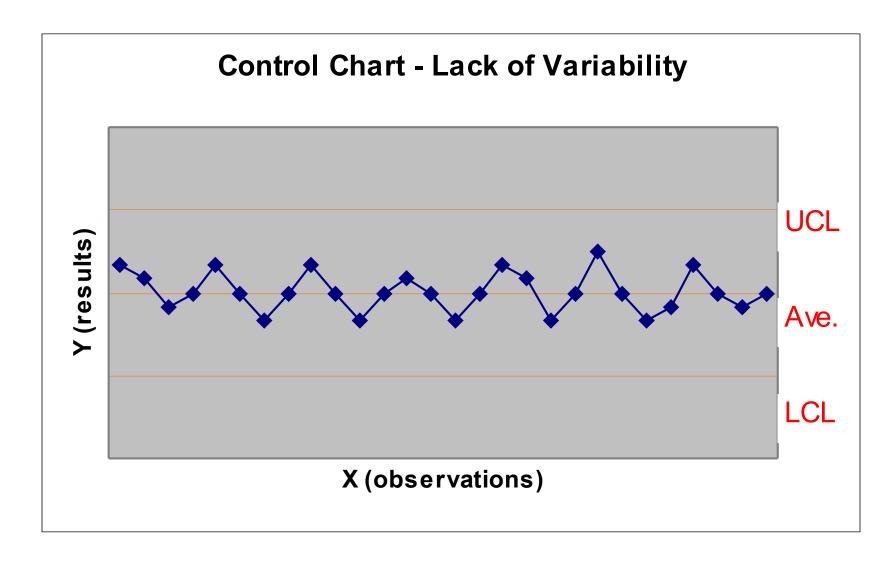
- SPECIAL: ANY POINT ABOVE UCL OR BELOW LCL
- RUN: ≥ 7 CONSECUTIVE PTS ABOVE OR BELOW CENTERLINE
- 1-IN-20: MORE THAN 1 POINT IN 20
   CONSECUTIVE POINTS CLOSE TO
   UCL OR LCL
- TREND: 5-7 CONSECUTIVE POINTS IN ONE DIRECTION (UP OR DOWN)

# CONTROL CHARTS IN CONTROL w/ CHANCE VARIATION CONTROL W/ CHANCE CONTROL CONTROL W/ CHANCE CONTROL W/ CHANCE CONTROL W/ CHANCE CONTROL W/ CHA



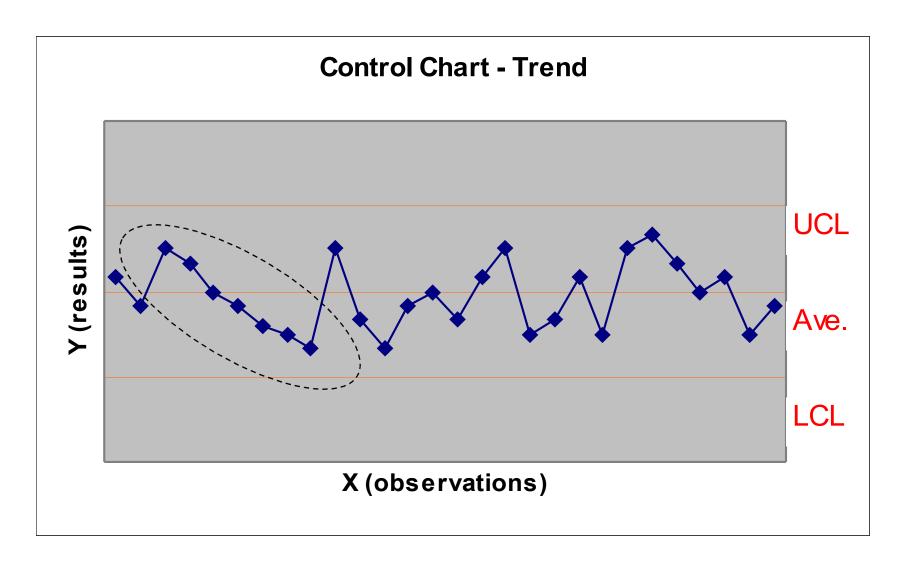
## CONTROL CHARTS LACK OF VARIABILITY





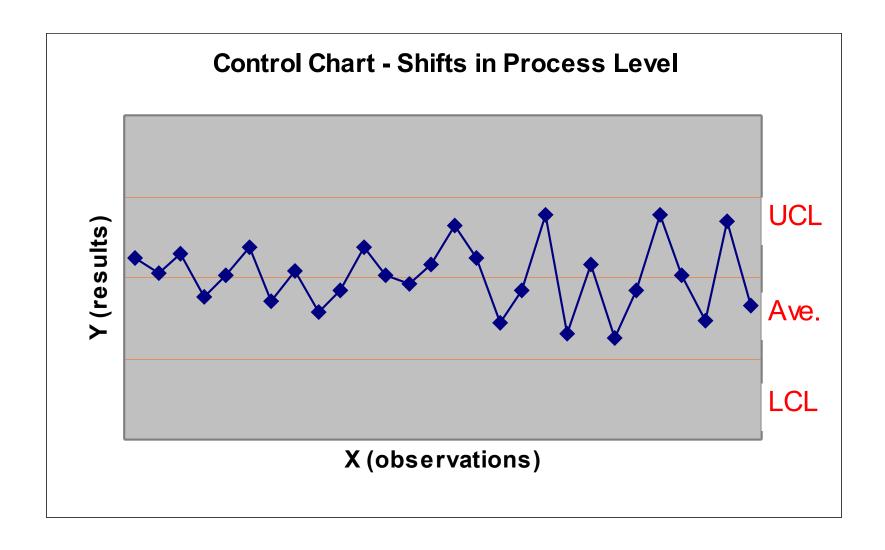
## CONTROL CHARTS TRENDS





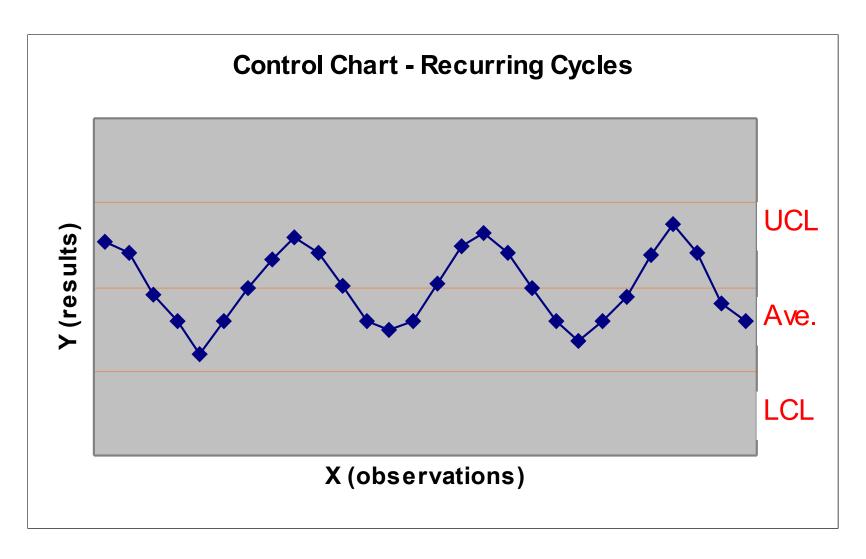
## CONTROL CHARTS SHIFTS IN PROCESS LEVELS





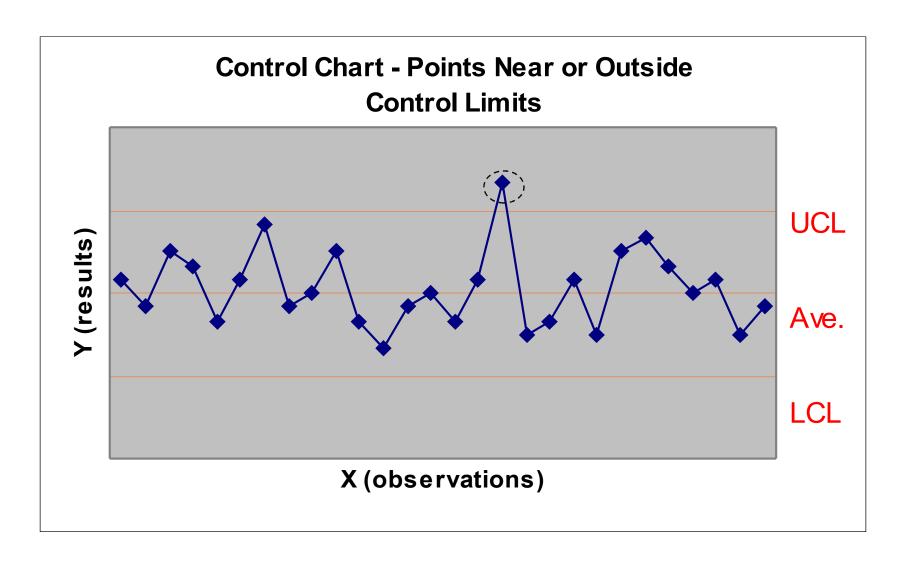
## **CONTROL CHARTS**RECURRING CYCLES





### **CONTROL CHARTS**POINTS NEAR OR OUTSIDE LIMITS





## CONTROL CHARTS ATTRIBUTE CHARTS



- TRACKS CHARACTERISTICS
  - SHORT OR TALL; PASS OR FAIL
- ONE CHART PER PROCESS
- FOLLOW TRENDS AND CYCLES
- EVALUATE ANY PROCESS CHANGE
- CONSISTS OF SEVERAL SUBGROUPS (a.k.a. - LOTS)
  - SUBGROUP SIZE ≥ 50

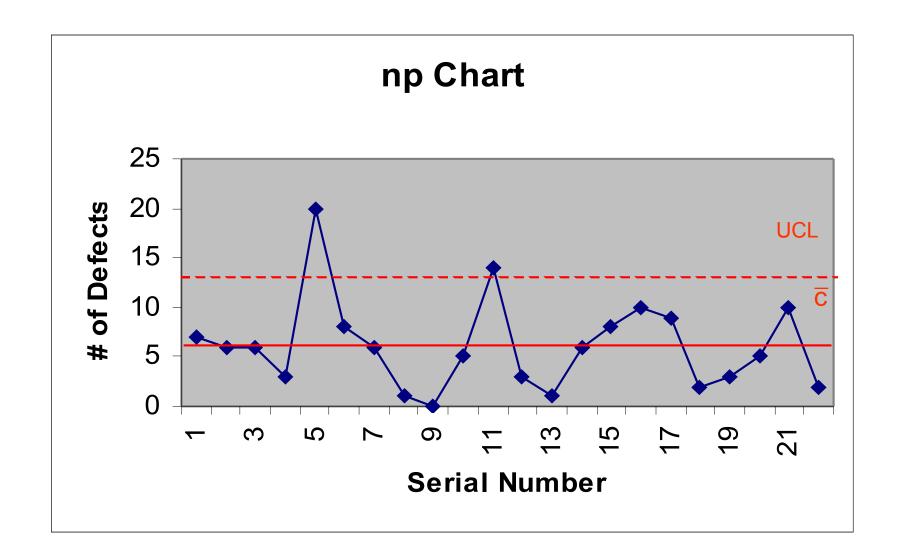
### CONTROL CHARTS ATTRIBUTE CHART TYPES



- p chart = Proportion Defective
- np chart = Number Defective
- c chart = Number of nonconformities within a constant sample size
- u chart = Number of nonconformities within a varying sample size

## CONTROL CHARTS np CHART EXAMPLE





## **CONTROL CHARTS**RISKS



- RISK 1: FALSE ALARM REJECT GOOD LOT CALL
   PROCESS OUT OF CONTROL WHEN IN CONTROL
- RISK 2: NO DETECTION OF PROBLEM

   SHIP BAD LOT
   CALL PROCESS IN CONTROL WHEN
   OUT OF CONTROL



Analysis for Extremence

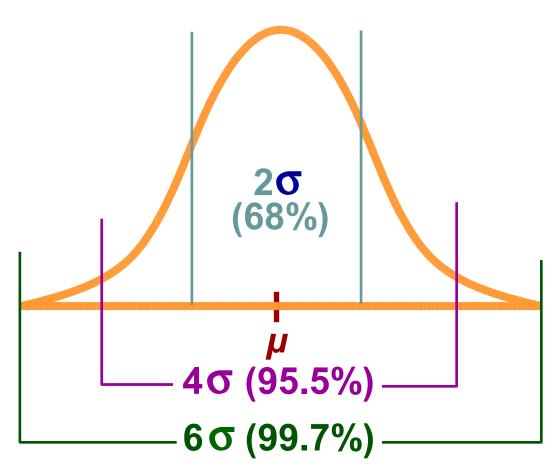
## **Process Capability Analysis**



- Differs Fundamentally from Control Charting
  - Focuses on improvement, not control
  - Variables, not attributes, data involved
  - Capability studies address range of individual outputs
  - Control charting addresses range of sample measures
- Assumes Normal Distribution
  - Remember the Empirical Rule?
  - Inherent capability  $(6\sigma_x)$  is compared to specifications
- Requires Process First to be In Control

# **Process Capability: Normal Curve**

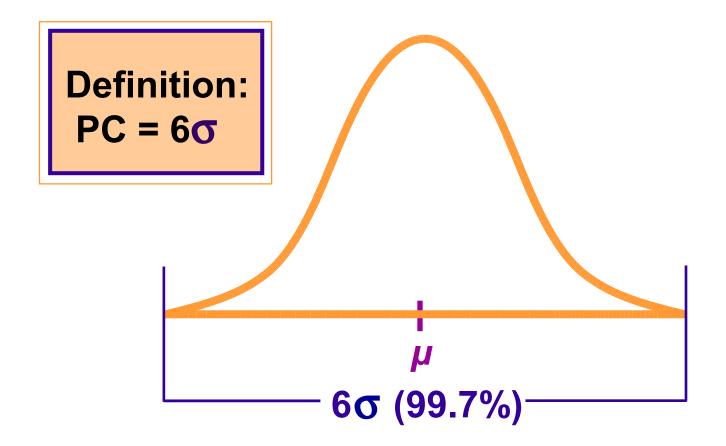






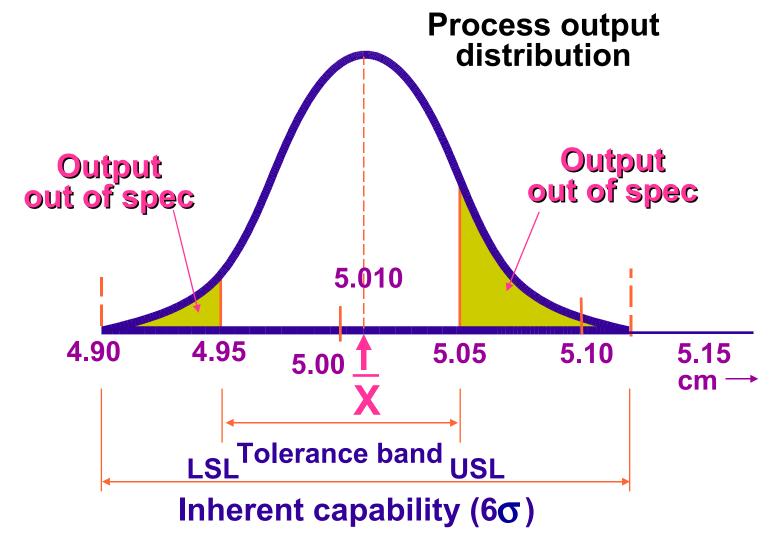


Process Capability (PC) is the range in which "all" output can be produced.

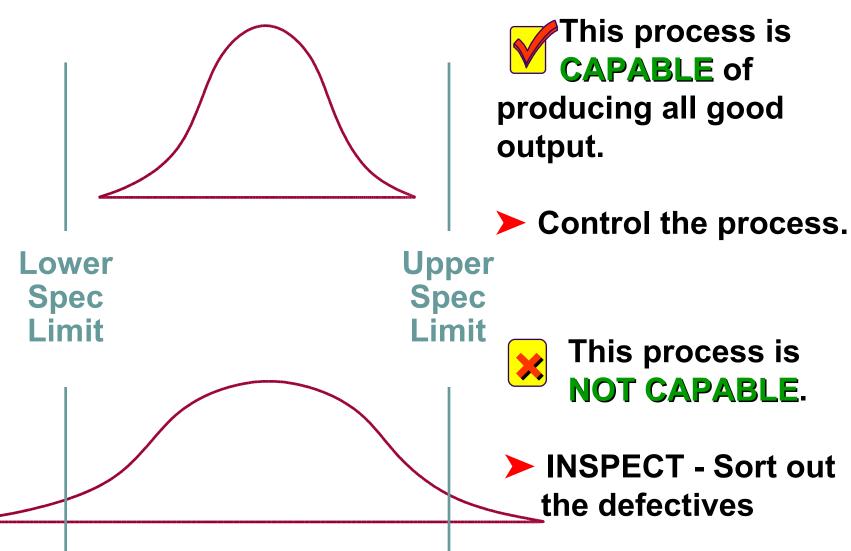














#### **Process Capability:**

Cp = Design Spec Width / Process Width

 $Cp = (USL-LSL) / 6\sigma$ 

Cp should be a large as possible

#### **Process Capability Ratio:**

Cr = 1/Cp \* 100

Indicates percent of design spec. used by process variability

Cr should be as small as possible



#### Process Capability Index (account for Mean Shifts):

$$Cpk = Cp * (1-k)$$

where k = Process Shift / (Design Spec Width/2)

Or

CpI = 
$$(\overline{X} - LSL)/3\sigma$$
  
Cpu =  $(USL - \overline{X})/3\sigma$ 

Cpu = 
$$(USL - X)/3\sigma$$



<u>Cpk</u> <u>Meaning</u>

Negative. Process Mean outside Spec Limits

0 - 1.0 Portion of process spread falls Outside Specs

> 1.0 Process spread falls within Spec Limits

Six Sigma Cpk = 1.5



Process Capability Ratio: Cp = Design Spec Width/Process Width

$$Cp = (USL-LSL)/6\sigma$$

**Process Capability Index (account for Mean Shifts):** 

$$Cpk = Cp (1-k)$$

where k = Process Shift/(Design Spec Width/2)

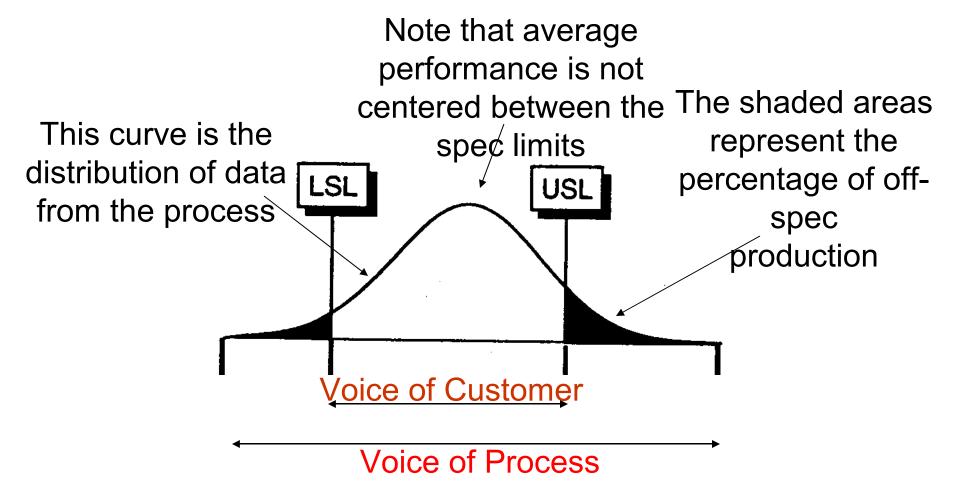
Or, Minimum of

- (X LSL)/3σ
- (USL X)/3σ



#### **Process Capability Ratios**

(Desired Performance) / (Actual Performance)





### Target rule:

**Cp - Cpk ≤ 0.33** 

Variation rule:

Cp ≥ 1.33

### **Process Capability Index**



Index C<sub>pk</sub> compares the spread and location of the process, relative to the specifications.

$$C_{pk} = \text{the smaller of } \begin{cases} OR & \frac{\text{Upper Spec Limit - }\overline{X}}{3\sigma} \\ & \frac{\overline{X} - \text{Lower Spec Limit}}{3\sigma} \end{cases}$$
Alternate Form

$$C_{pk} = \frac{Z_{min}}{3}$$
Where  $Z_{min}$  is the smaller of: 
$$\frac{Upper Spec Limit - \bar{X}}{\sigma}$$

$$\frac{\bar{X} - Lower Spec Limit}{\sigma}$$

#### **Process Capability: Cpk Variations**



(a)

(b)

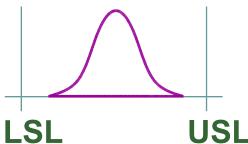
(c)

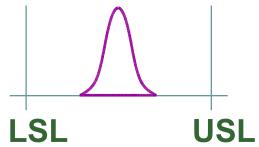
$$C_{pk} = 1.33$$









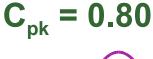


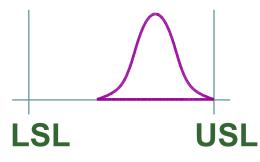
(e)

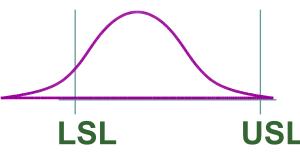
**(f)** 

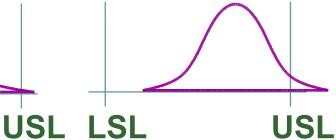
$$C_{pk} = 1.0$$











#### PROCESS CAPABILITY MEASUREMENT

#### **Process Capability is computed as:**

$$6\sigma = 6S = 6R/d$$

Process Capability Index  $Cp = U - L / 6 \sigma$ 

Cpk = 
$$U - X/3\sigma$$

If: Cp > 1.6

Cp > 1.3

Cp > 1.0

Cp < 1.0

**Process is Excellent** 

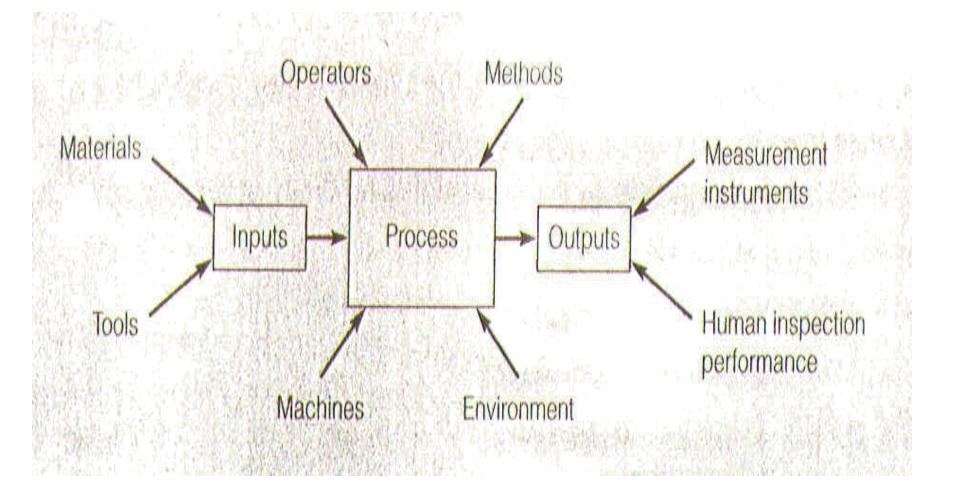
**Process is Good** 

**Process is Satisfactory** 

**Process is Poor** 

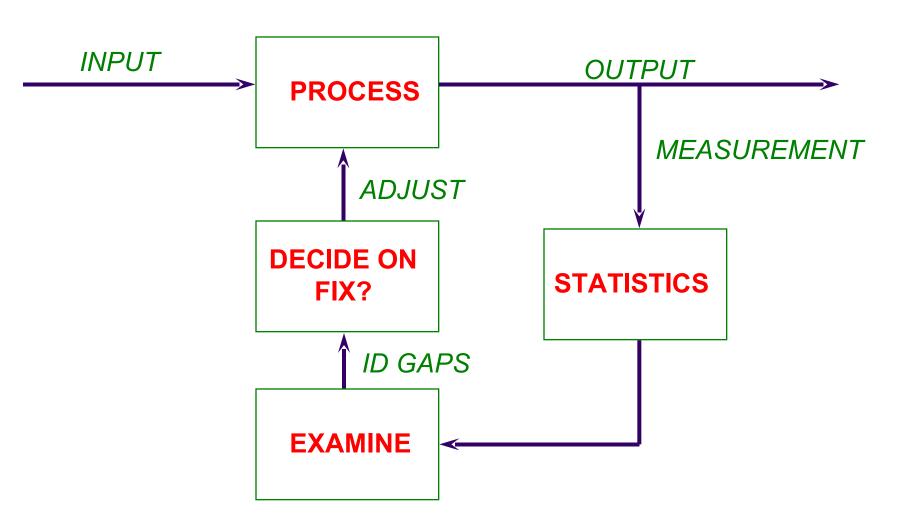
## Sources of Variation in Production Processes





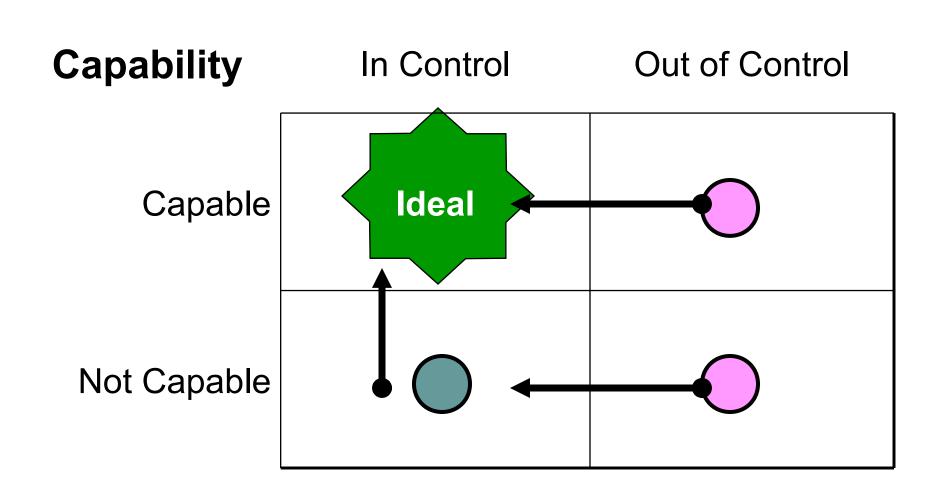
#### **CONTROL LOOP**







#### **Control**





#### **Contents**

- Quality & TQM
- Basic Statistics
- Seven QC Tools
- Control Charts
- Process Capability Analysis



## Thank You.